Asteroid lightcurve inversion using Lommel-Seeliger ellipsoids

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The rotational period, pole orientation, and convex three-dimensional shape of an asteroid can be derived from photometric lightcurves observed in a number of apparitions with varying illumination and observation geometries (e.g., Kaasalainen *et al.* 2001, Torppa *et al.* 2008, Durech *et al.* 2009). It is customary to estimate the rotational period with a simplified shape model and a small number of trial pole orientations. Once the period is available, the pole orientation can be refined with a general convex shape model represented by the spherical harmonics expansion for the Gaussian surface density. Once the Gaussian surface density is available, the actual convex shape is constructed as a solution of the Minkowski problem.

We focus on the initial derivation of the rotational period and pole orientation with the help of the Lommel-Seeliger ellipsoid (LS-ellipsoid), a triaxial ellipsoid with a Lommel-Seeliger surface scattering law. The disk-integrated photometric brightness for the LS-ellipsoid is available in a closed form (Muinonen and Lumme, in preparation), warranting efficient direct computation of lightcurves.

With modern computers and the LS-ellipsoid, the rotation period, pole orientation, and ellipsoidal shape can be derived, in principle, simultaneously (see Cellino *et al.*, present meeting). However, here we choose to proceed systematically as follows. First, the rotation period is scanned systematically across its relevant range with a resolution of $P_0^2/2T$ dictated by a tentative period estimate P_0 and the time interval spanned by the photometric data T. This is typically carried out for a small number of pole orientations distributed uniformly on a unit sphere. For each pole orientation, the ellipsoid pole orientation, rotational phase, and axial ratios are optimized with the help of the Nelder-Mead downhill simplex method. Although the shape optimization can suffer from getting stuck in local minima, overall, the rotation period is fairly accurately obtained by the initial scanning.

Second, for the rotation period obtained, the pole orientation can be mapped with a high resolution pertaining to a few degrees on the unit sphere, with downhill simplex optimization for the rotation period, rotational phase, and axial ratios in the case of each trial pole orientation. Third, after mapping the pole orientation, the regimes of minima are evident, and analyses can be focused on each of the regimes separately. Now all the parameters can be optimized to obtain the best single fit to the data.

Next, in order to allow for an efficient Markov-chain Monte Carlo analysis (MCMC) in the neighborhood of the best-fit solution, we generate virtual observations, by adding random noise to the observations, and repeat the optimization for the parameters by using the virtual data (see Wang and Muinonen, present meeting). The differences of the virtual best-fit parameters can then be utilized as proposals in the final MCMC sampling of the parameters. Finally, at the end, we obtain a description of the probability density function for the period, pole, rotational phase, and ellipsoid axial ratios in the neighborhood of the best-fit parameters. The MCMC analysis can be repeated for each different solution regime separately and mutual significance of the separate regimes can be assessed. Note that uniform sampling of the phase space can be carried out in a way similar to that in asteroid orbital inversion (see Muinonen *et al.*, present meeting).

We apply the tools to both sparse (Hipparcos and forecoming Gaia data; Cellino *et al.* 2009; Cellino *et al.*, present meeting) and dense photometric data (traditional lightcurve data; Wang and Muinonen, present meeting).

References: Cellino, A., Hestroffer, D., Tanga, P., Mottola, S., and Dell'Oro, A. (2009). Genetic inversion of sparse disk-integrated photometric data of asteroids: application to Hipparcos data. *Astron. Astrophys.* **506**, 935; Kaasalainen, M., Torppa, J., and Muinonen, K. (2001). Optimization methods for asteroid lightcurve inversion. II. The complete inverse problem. *Icarus* **153**, 37; Durech, J., Kaasalainen, M., Warner, B. D., Fauerbach, M., Marks, S. A., Fauvaud, S., Fauvaud, M., Vugnon, J.-M., Pilcher, F., Bernasconi, L., and Behrend, R. (2009). Asteroid models from combined sparse and dense photometric data. *Astron. Astrophys.* **493**, 291; Torppa, J., Hentunen, V-P., Paakkonen, P., Kehusmaa, P., and Muinonen, K. (2008). Asteroid shape and spin statistics from convex models. *Icarus* **198**, 91.