The H,G_1,G_2 photometric system with scarce observational data

A. Penttilä[1], M. Granvik[1,2], K. Muinonen[1,2], and O. Wilkman[1]

[1]Department of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland
[2]Finnish Geodetic Institute, P.O. Box 15, FI-02431 Masala, Finland

The H,G_1,G_2 photometric system was officially adopted at the IAU General Assembly in Beijing, 2012. The system replaced the H,G system from 1985. The 'photometric system' is a parametrized model $V(\alpha;\text{params})$ for the magnitude-phase relation of small Solar System bodies, and the main purpose is to predict the magnitude at backscattering, $H := V(0°)$, i.e., the (absolute) magnitude of the object. The original H,G system was designed using the best available data in 1985, but since then new observations have been made showing certain features, especially near backscattering, to which the H,G function has troubles adjusting to. The H,G_1,G_2 system was developed especially to address these issues [1].

With a sufficient number of high-accuracy observations and with a wide phase-angle coverage, the H,G_1,G_2 system performs well. However, with scarce low-accuracy data the system has troubles producing a reliable fit, as would any other three-parameter nonlinear function. Therefore, simultaneously with the H,G_1,G_2 system, a two-parameter version of the model, the H,G_{12} system, was introduced [1]. The two-parameter version ties the parameters G_1,G_2 into a single parameter G_{12} by a linear relation, and still uses the H,G_1,G_2 system in the background. This version dramatically improves the possibility to receive a reliable phase-curve fit to scarce data.

The amount of observed small bodies is increasing all the time, and so is the need to produce estimates for the absolute magnitude/diameter/albedo and other size/composition related parameters. The lack of small-phase-angle observations is especially topical for near-Earth objects (NEOs). With these, even the two- parameter version faces problems. The previous procedure with the H,G system in such circumstances has been that the G-parameter has been fixed to some constant value, thus only fitting a single-parameter function.

In conclusion, there is a definitive need for a reliable procedure to produce photometric fits to very scarce and low-accuracy data. There are a few details that should be considered with the H,G_1,G_2 or H,G_{12} systems with scarce data. The first point is the distribution of errors in the fit. The original H,G system allowed linear regression in the flux space, thus making the estimation computationally easier. The same principle was repeated with the H,G_1,G_2 system. There is, however, a major hidden assumption in the transformation. With regression modeling, the residuals should be distributed symmetrically around zero. If they are normally distributed, even better. We have noticed that, at least with some NEO observations, the residuals in the flux space are far from symmetric, and seem to be much more symmetric in the magnitude space. The result is that the nonlinear fit in magnitude space is far more reliable than the linear fit in the flux space. Since the computers and nonlinear regression algorithms are efficient enough, we conclude that, in many cases, with low-accuracy data the nonlinear fit should be favored.

In fact, there are statistical procedures that should be employed with the photometric fit. At the moment, the choice between the three-parameter and two-parameter versions is simply based on subjective decision-making. By checking parameter error and model comparison statistics, the choice could be done objectively. Similarly, the choice between the linear fit in flux space and the nonlinear fit in magnitude space should be based on a statistical test of unbiased residuals. Furthermore, the so-called Box-Cox transform could be employed to find an optimal transformation somewhere between the magnitude and flux spaces.

The H,G_1,G_2 system is based on cubic splines, and is therefore a bit more complicated to implement than a system with simpler basis functions. The same applies to a complete program that would automatically choose the best transforms to data, test if two- or three-parameter version of the model should be fitted, and produce the fitted parameters with their error estimates. Our group has already made implementations of the H,G_1,G_2 system publicly available [2]. We plan to implement the abovementioned improvements to the system and make also these tools public.

References: [1] K. Muinonen, I. N. Belskaya, A. Cellino, M. Delbò, A.-C. Levasseur-Regourd, A. Penttilä, and E. F. Tedesco (2010). A three-parameter magnitude phase function for asteroids. *Icarus* **209**(2), 542–555. [2] A. Penttilä (2012). Collection of tools for H,G1,G2 system, http://wiki.helsinki.fi/display/PSR/HG1G2+tools.