Monte Carlo solution of the volume-integral equation of electromagnetic scattering

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Electromagnetic scattering is often the main physical process to be understood when interpreting the observations of asteroids, comets, and meteors. Modeling the scattering faces still many problems, and one needs to assess several different cases: multiple scattering and shadowing by the rough surface, multiple scattering inside a surface element, and single scattering by a small object.

Our specific goal is to extend the electromagnetic techniques to larger and more complicated objects, and derive approximations taking into account the most important effects of waves. Here we experiment with Monte Carlo techniques: can they provide something new to solving the scattering problems?

The electromagnetic wave equation in the presence of a scatterer of volume V and refractive index m, with an incident wave $\mathbf{E}_0$, including boundary conditions and the scattering condition at infinity, can be presented in the form of an integral equation

$$\mathbf{E}(\mathbf{r})(1+\chi(\mathbf{r})Q(\rho)) - \int_{V-V_\rho} \mathrm{d}\mathbf{r}'\mathsf{G}(\mathbf{r}-\mathbf{r}')\chi(\mathbf{r}')\mathbf{E}(\mathbf{r}') = \mathbf{E}_0, \tag{1}$$

where $\chi(\mathbf{r}) = m(\mathbf{r})^2 - 1$, $Q(\rho) = -1/3 + \mathcal{O}(\rho^2) + \mathcal{O}'(m^2\rho^2)$, $\mathcal{O}$, and $\mathcal{O}'$ are some second- and higher-order corrections for the finite-size volume V_ρ of radius ρ around the singularity and G is the dyadic Green's function of the form

$$\mathsf{G}(\mathbf{R}) = \frac{\exp(\mathrm{i}kR)}{4\pi R}\left[1\left(1+\frac{\mathrm{i}}{R}-\frac{1}{R^2}\right) - \mathbf{RR}\left(1+\frac{3\mathrm{i}}{R}-\frac{3}{R^2}\right)\right]. \tag{2}$$

In general, this is solved by extending the internal field in terms of some simple basis functions, e.g., plane or spherical waves or a cubic grid, approximating the integrals in a clever way, and determining the goodness of the solution somehow, e.g., moments or least square. Whatever the choice, the solution usually converges nicely towards a correct enough solution when the scatterer is small and simple, and diverges when the scatterer becomes too complicated. With certain methods, one can reach larger scatterers faster, but the memory and CPU needs can be huge.

Until today, all successful solutions are based on more or less regular quadratures. Because of the oscillating singularity of the Green's function, the quadrature must match exactly the canceling patterns of the integrand, and any improper quadrature leads to large errors. Monte Carlo based integration appears thus a very bad choice, but we take the challenge, and formulate the integration applying a three-finger rule to catch the singularity. Our other selections are the least-squares technique and plane-wave basis, though both can be freely and easily changed. The singularity is treated fully numerically, and the radius ρ is assumed so small that the correction terms do not contribute. Any other choice only worsens the accuracy, without a significant gain in speed.

As with any other technique, we can solve small spheres of size $x < 5/|m|$ within an hour of processor time with about 1% accuracy for a large range of refractive indices. In speed, this technique does not compete with faster techniques such as ADDA, but in some random cases the accuracy can be even better (probably due to sub-optimal singularity formula in ADDA — applying numerical integration also there could probably make ADDA winner in all the cases). We continue towards more complicated cases and multiple scattering to see, if some further improvements can be made.

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