Discrete exterior calculus for numerical simulation of meteor head-echo radar reflections

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The meteor head-echo feature has been studied by high-power large-aperture (HPLA) radars since 1960's (see Evans 1965). Based on the observations conducted by the different radar systems and post-processing techniques, there exist several models for the meteor head-echo simulations. One reason for this is the characteristics of the radar system, e.g., in terms of frequency and antenna geometry (see Kero et al. 2012). It is also worth mentioning that there are significant differences in the meteor sizes. According to the observations reported by, e.g., Vertatschitsch et al. (2011) and Wannberg et al. (2011), the head echo can be modeled as overdense scatter from a plasma layer, surrounding the meteor, with a certain density distribution. In these models, the plasmatic object is assumed to be a conducting spherical object, and the electromagnetic phenomenon can be presented by partial differential equations coupling the electric and magnetic fields.

The traditional way of solving electromagnetic problems presented in space-time-domain as partial differential equations is to use the finite-difference time-domain method (FDTD; see Dyrud et al. 2008). In this study, we use more generalized finite differences by applying the discrete exterior calculus (DEC) to the numerical simulation of meteor head-echo radar reflections. The properties and calculus of differential forms is provided in a natural way at the discretization stage, and we associate the degrees of freedom of the electric and magnetic fields to the primal and dual mesh structures, respectively. The connection between the primal and dual forms is obtained by the discrete Hodge operator, the quality of which depends on the mesh construction. Our generalized formulation of the DEC for the Maxwell equations (see Pauly and Rossi 2011) works basically on unstructured grids, and it covers both the classical Yee's FDTD scheme and the Bossavit-Kettunen approach (Bossavit and Kettunen 1999). The method has been shown to give promising results with both time-dependent (Räbinä et al. 2014a) and time-harmonic problems (Räbinä et al. 2014b).

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