Asteroid orbital inversion using uniform phase-space sampling

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We review statistical inverse methods for asteroid orbit computation from a small number of astrometric observations and short time intervals of observations. With the help of Markov-chain Monte Carlo methods (MCMC), we present a novel inverse method that utilizes uniform sampling of the phase space for the orbital elements.

The statistical orbital ranging method (Virtanen *et al.* 2001, Muinonen *et al.* 2001) was set out to resolve the long-lasting challenges in the initial computation of orbits for asteroids. The ranging method starts from the selection of a pair of astrometric observations. Thereafter, the topocentric ranges and angular deviations in R.A. and Decl. are randomly sampled. The two Cartesian positions allow for the computation of orbital elements and, subsequently, the computation of ephemerides for the observation dates. Candidate orbital elements are included in the sample of accepted elements if the χ^2-value between the observed and computed observations is within a pre-defined threshold. The sample orbital elements obtain weights based on a certain debiasing procedure. When the weights are available, the full sample of orbital elements allows the probabilistic assessments for, e.g., object classification and ephemeris computation as well as the computation of collision probabilities.

The MCMC ranging method (Oszkiewicz *et al.* 2009; see also Granvik *et al.* 2009) replaces the original sampling algorithm described above with a proposal probability density function (p.d.f.), and a chain of sample orbital elements results in the phase space. MCMC ranging is based on a bivariate Gaussian p.d.f. for the topocentric ranges, and allows for the sampling to focus on the phase-space domain with most of the probability mass.

In the virtual-observation MCMC method (Muinonen *et al.* 2012), the proposal p.d.f. for the orbital elements is chosen to mimic the a posteriori p.d.f. for the elements: first, random errors are simulated for each observation, resulting in a set of virtual observations; second, corresponding virtual least-squares orbital elements are derived using the Nelder-Mead downhill simplex method; third, repeating the procedure two times allows for a computation of a difference for two sets of virtual orbital elements; and, fourth, this orbital-element difference constitutes a symmetric proposal in a random-walk Metropolis-Hastings algorithm, avoiding the explicit computation of the proposal p.d.f. In a discrete approximation, the allowed proposals coincide with the differences that are based on a large number of pre-computed sets of virtual least-squares orbital elements. The virtual-observation MCMC method is thus based on the characterization of the relevant volume in the orbital-element phase space.

Here we utilize MCMC to map the phase-space domain of acceptable solutions. We can make use of the proposal p.d.f.s from the MCMC ranging and virtual-observation methods. The present phase-space mapping produces, upon convergence, a uniform sampling of the solution space within a pre-defined χ^2-value. The weights of the sampled orbital elements are then computed on the basis of the corresponding χ^2-values. The present method resembles the original ranging method. On one hand, MCMC mapping is insensitive to local extrema in the phase space and efficiently maps the solution space. This is somewhat contrary to the MCMC methods described above. On the other hand, MCMC mapping can suffer from producing a small number of sample elements with small χ^2-values, in resemblance to the original ranging method.

We apply the methods to example near-Earth, main-belt, and transneptunian objects, and highlight the utilization of the methods in the data processing and analysis pipeline of the ESA Gaia space mission.

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