# Fast method for the estimation of impact probability of near-Earth objects

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We propose a method to estimate the probability of collision of a celestial body with the Earth (or another major planet) at a given time moment $t$. Let there be a set of observations of a small body. At initial time moment $T_0$, a nominal orbit is defined by the least squares method. In our method, a unique coordinate system is used. It is supposed that errors of observations are related to errors of coordinates and velocities linearly and the distribution law of observation errors is normal.

The unique frame is defined as follows. First of all, we fix an osculating ellipse of the body's orbit at the time moment $t$. The mean anomaly $M$ in this osculating ellipse is a coordinate of the introduced system. The spatial coordinate $\xi$ is perpendicular to the plane which contains the fixed ellipse. $\eta$ is a spatial coordinate, too, and our axes satisfy the right-hand rule. The origin of $\xi$ and $\eta$ corresponds to the given $M$ point on the ellipse. The components of the velocity are the corresponding derivatives of $\dot{\xi}, \dot{\eta}, \dot{M}$.

To calculate the probability of collision, we numerically integrate equations of an asteroid's motion taking into account perturbations and calculate a normal matrix $N$. The probability is determinated as follows:

$$P = \frac{|detN|^{\frac{1}{2}}}{(2\pi)^3} \int_{\Omega} e^{-\frac{1}{2}x^T N x} dx$$

where $x$ denotes a six-dimensional vector of coordinates and velocities, $\Omega$ is the region which is occupied by the Earth, and the superscript $T$ denotes the matrix transpose operation. To take into account a gravitational attraction of the Earth, the radius of the Earth is increased by $\sqrt{1+\frac{v_s^2}{v_{rel}^2}}$ times, where $v_s$ is the escape velocity and $v_{rel}$ is the small body's velocity relative to the Earth.

The 6-dimensional integral is analytically integrated over the velocity components on $(-\infty, +\infty)$. After that we have the $3\times3$ matrix $N_1$. That 6-dimensional integral becomes a 3-dimensional integral. To calculate it quickly we do the following. We introduce a new coordinate system $(\xi, \eta, \frac{M}{d})$, where $d$ denotes $d = \frac{\dot{M}}{|V|}$. It is supposed that $\Omega$ is a full-sphere in the coordinate system $(\xi, \eta, \frac{M}{d})$. Using a singular decomposition of the matrix $N_1$ we obtain 3 one-dimensional integrals instead of the three-dimensional one. It allows to decreasing of several orders the time of calculations.

The impact probabilities for 8 asteroids were calculated by the proposed method in the introduced coordinate system $(\xi, \eta, M)$. Also, we calculated it using the above described scheme in the Cartesian coordinate system. To check the accuracy of the obtained values, the probabilities were computed by a Monte Carlo method, too. The error of probability calculation by Monte Carlo is estimated according to $\sigma_{MC} \approx \frac{\sqrt{P_{MC}}}{\sqrt{n}}$, where $n$ is a number of realizations, and $P_{MC}$ is a probability. Results of calculations are given in the table below, where the "object" is an asteroid designation, $t$ denotes a time of collision, and $P_{\xi,\eta,M}$ and $P_{x,y,z}$ are probabilities which were calculated by the proposed method in the coordinate system $(\xi, \eta, M)$ and in the Cartesian coordinate system, respectively.

| Object | $t$ | $P_{\xi,\eta,M}$ | $P_{MC}$ | $3\sigma_{MC}$ | $P_{x,y,z}$ |
|---|---|---|---|---|---|
| 2009 JF1 | 2022.05.06,34 | $6.6 \cdot 10^{-4}$ | $7.4 \cdot 10^{-4}$ | $1.2 \cdot 10^{-4}$ | $6.7 \cdot 10^{-4}$ |
| 2005 BS1 | 2016.01.14,44 | $1.5 \cdot 10^{-4}$ | $1.4 \cdot 10^{-4}$ | $2.4 \cdot 10^{-5}$ | 0 |
| 2006 QV89 | 2019.09.09,38 | $2.2 \cdot 10^{-3}$ | $1.8 \cdot 10^{-3}$ | $1.0 \cdot 10^{-4}$ | $2.5 \cdot 10^{-3}$ |
| 2007 VK184 | 2048.06.03,09 | $3.0 \cdot 10^{-5}$ | $6.2 \cdot 10^{-6}$ | $2.0 \cdot 10^{-6}$ | $2.9 \cdot 10^{-5}$ |
| 2008 CK70 | 2030.02.14,67 | $6.4 \cdot 10^{-4}$ | $6.4 \cdot 10^{-4}$ | $9.6 \cdot 10^{-5}$ | $6.7 \cdot 10^{-4}$ |
| 2005 QK76 | 2030.02.26,32 | $3.8 \cdot 10^{-5}$ | $4.3 \cdot 10^{-5}$ | $8.5 \cdot 10^{-6}$ | 0 |
| 2008 JL3 | 2027.05.01,41 | $4.7 \cdot 10^{-4}$ | $3.0 \cdot 10^{-4}$ | $4.3 \cdot 10^{-5}$ | $4.7 \cdot 10^{-4}$ |
| 2007 KO4 | 2015.11.23,20 | $4.0 \cdot 10^{-7}$ | $7.3 \cdot 10^{-7}$ | $4.0 \cdot 10^{-7}$ | 0 |

The table shows that the probabilities $P_{\xi,\eta,M}$ are close to the probabilities obtained by Monte Carlo method, but require much less time of computation (e.g., 4 millions times less for 2007 $KO_4$, and takes about 2 seconds using a processor Intel Core i7-2600 3.40GHz). Utilization of the new coordinate system, according to a nominal orbit, gives better results than the utilization of the Cartesian system.