Numerical simulations of light scattering characteristics of ice fractal particles

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For the model of a vertically and horizontally homogeneous slab of an arbitrary optical thickness composed of fractal aggregates of small spherical ice monomers, generated by applying three different approaches, we analyze the results of computations of the backscattering circular polarization ratio. The computations are performed for the refractive index $m = 1.78 + i0.003$ by using the superposition T-matrix and vector radiative transfer codes.

INTRODUCTION

Scattering of electromagnetic waves by media composed of ice fractal particles is the subject of utmost importance to the discipline of remote sensing of the Earth and other solar system objects. In particular, the knowledge of scattering properties of such media is needed for the interpretation of radar observations of terrestrial ice sheets [1], Galilean satellites of Jupiter [2], and Saturn's rings [3]. To solve this problem it is necessary, in the first place, to calculate the single scattering characteristics of fractal aggregate particles, and various techniques have been developed for this purpose over many years (see [4]). In recent years, the numerically-exact superposition T-matrix method [5] has been extensively applied to the computations of scattering by multi-sphere clusters (see, e.g., [6]). It is obvious that when performing such computations the model of aggregate structure must be adopted, and it should be expected that scattering characteristics of clusters can be affected in some way by their morphology. The question is that of how strong these effects can be.

In this paper, for a slab composed of fractal aggregates of small spherical ice monomers, generated by applying several procedures, we investigate the influence of the cluster morphology on the behavior of the circular backscattering polarization ratio which may contain information on microphysical properties of the scattering medium [3].

ICE CLUSTER MODELS AND COMPUTATIONAL TECHNIQUES

Let the scattering medium be a plane-parallel slab composed of randomly distributed, independently scattering aggregate particles built of spherical monomers. The slab is illuminated

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by a parallel beam of light, with (θ_0, φ_0) and (θ, φ) specifying the directions of incidence and reflection. The first step in computations of the single-scattering characteristics of the medium composed of fractal-like clusters is generating monomer positions in a fractal aggregate. As an initial model of aggregates we adopt fractal clusters which can be described by the following statistical scaling law [4]:

$$N_{\mathrm{S}} = k_0 \left(\frac{R_g}{r} \right)^{D_{\mathrm{f}}}, \tag{1}$$

where r is the monomer radius, $1 \leq D_{\mathrm{f}} \leq 3$ is the fractal dimension, k_0 is the fractal prefactor, N_{S} is the number of monomers in the aggregate, and R_{g}, called the radius of gyration, is a measure of the overall aggregate radius. Parameters N_{S}, D_{f}, and k_0 specify the morphology of a fractal aggregate.

We apply the following algorithms to generate monomer positions in fractal aggregates.

A) The cluster–cluster aggregation procedure [7]. The basic idea of this method is to generate a sequence of random sphere positions subject to the constraint that the positions, at any point in the sequence, identically satisfy Eq. (1) for given k_0 and D_{f} and that each monomer touches at least one other monomer.

B) The original diffusion-limited aggregation (DLA) method [8] in which the generation procedure starts with a pair of spheres in contact for pre-set k_0 and D_{f} values and adds a single monomer at a time. Compared to the cluster–cluster aggregation procedure, the DLA algorithm is less realistic, but it can be used for a wider range of the values of the fractal parameters. Note that the overall shape of fractals generated by using both the cluster-cluster and DLA algorithms is nearly spherical.

C) The approach which gives the possibility to generate clusters of spheroidal overall shape. Specifically, it generates fractals composed of spherical monomers that lie randomly within the surface of a spheroid, subject to the constraints that no two spheres overlap and that each sphere contacts with at least one neighbour. In the case of unit monomers, the monomer packing density p is defined as

$$p = \frac{N_S}{ER^3}, \tag{2}$$

where E is the aspect ratio, R is the axial radius of the spheroid, and $E > 1$ represents a prolate spheroid. Note that such fractals are not described by Eq. (1).

To compute the elements of the single-scattering matrix for the generated fractals, we use the superposition T-matrix method [5] and the corresponding FORTRAN code which is publicly available online*. Then the obtained single-scattering characteristics of aggregate particles are used to compute the elements of the diffuse Stokes reflection matrix. In the case of a semi-infinite homogeneous slab we employ a vector radiative-transfer code based on the numerical solution of the Ambarzumian's nonlinear integral equation [9], and for a finite slab

* ftp://ftp.eng.aubum.edu/pub/dmckwski/scatcodes/index.html

we use a numerically exact computer code based on the invariant imbedding technique [10]. The circular polarization ratio is derived from Eqs. (14.3.21)–(14.3.25) and (14.5.15) of [11].

NUMERICAL RESULTS AND DISCUSSION

In Fig. 1, we depict the computed dependences of the backscattering circular polarization ratio μ_C on the angle $B = \pi/2 - \theta$. The computations are performed for a plane-parallel homogeneous slab of optical thickness $\tau = 2$ (three bottom curves) and a semi-infinite slab (three top curves) composed of fractal ice aggregates with the refractive index $m = 1.78 + i0.003$ which corresponds to weakly contaminated water ice at a wavelength of 12.6 cm. The monomer packing density p is fixed at 0.2, the monomer radius $r = 1$ cm, the fractal dimension $D_f = 2.5$, and the overall cluster radius R is varied in the range $4 \le R \le 10$ cm. Similarly [12], to obtain the single-scattering characteristics, we generate an ensemble of ten fractal-parameter-equivalent realizations of an aggregate, compute the single-scattering characteristics for each realization, and then average them over the ensemble. The necessity to perform such averaging was discussed in [12].

The top row shows the results of computations performed for slabs composed of aggregates generated by employing the three procedures described above. First of all, we see that for $\tau = 2$, the angular dependence of μ_C on B decreases significantly with increasing R, but it increases with increasing τ. One can see that when using the cluster-cluster and DLA algorithms, for the same values of τ the values of μ_C do not practically depend on the aggre-

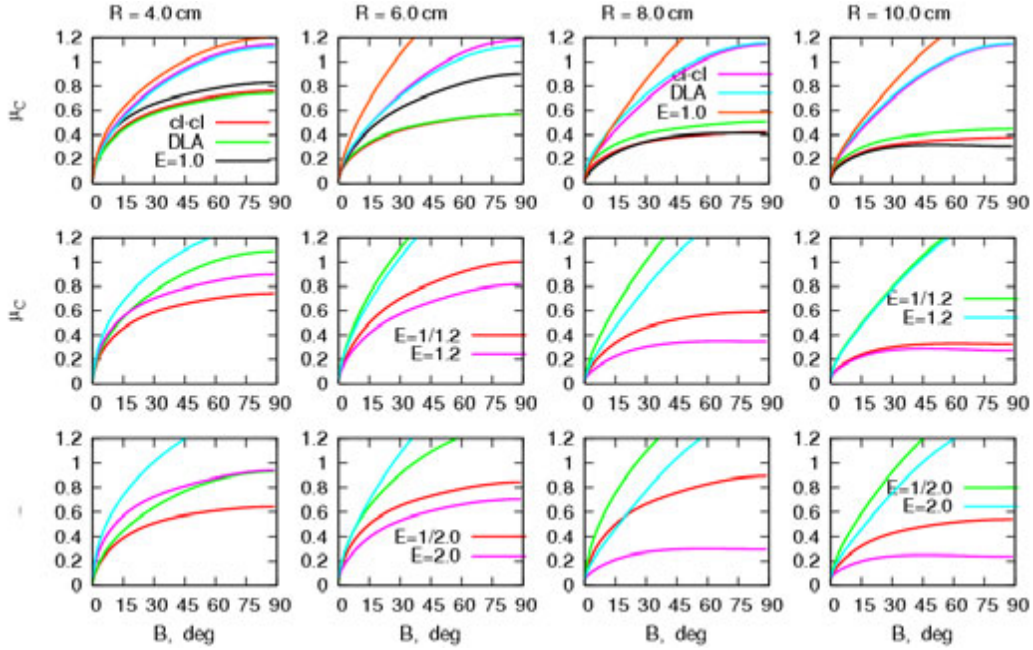

Figure 1. Angular dependence of the backscattering circular polarization ratio. Different curves correspond to the results of computations performed for a layer of two values of the optical thickness (τ = 2 and semi-infinite) and varying cluster structure.

gate generating procedure, the dependence increases weakly with increasing R. Our computations performed for $D_f = 3$ (not presented here) show that the difference in the values of μ_C due to the difference in the aggregate structure produced by employing such procedures, substantially increases with increasing D_f. As regards the case of clusters described by Eq. (2) for E = 1, $\tau = 2$, $R = 4$, 6 cm, a significant difference in the values of μ_C obtained by using this approach and the cluster-cluster or the DLA procedures is seen. This difference increases with increasing R from 4 cm up to 6 cm, but with further increasing R it starts to decrease noticeably. The numerical results obtained for aggregates of prolate (E > 1) and oblate (E < 1) spheroidal overall shapes are depicted in the middle ($E = 1.2, 1/1.2$) and bottom ($E = 2, 1/2$) rows. One can see that with the exception of the case of $R = 10$ cm, $E = 1.2, 1/1.2$, for the slabs composed of fractals of the overall oblate and prolate spheroidal shapes, the values of μ_C differ from each other.

Thus, the results of our computations show that, for the slabs composed of aggregates generated by the cluster-cluster and the DLA procedure, the values of the backscattering polarization ratio differ from each other very weakly. In the case of using Eq. (2), the dependence of μ_C on aggregate structure can be significant.

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