Comparison of different TIRM schemes based on the DSM

N. Grishina*,[1], E. Eremina[2], Y. Eremin[1], and T. Wriedt[3]

[1]*Moscow Lomonosov State University, Vorobyov's Hills, 119991 Moscow, Russia.*
[2]*University of Bremen, Badgasteiner str.3, 28359 Bremen, Germany.*
[3]*Institute of Materials' science, Badgasteiner str.3, 28359 Bremen, Germany.*

Total Internal Reflection Microscopy (TIRM) is an effective technique to measure weak interactions between colloidal particles in solution and a flat surface. In this paper, the Discrete Sources Method (DSM) has been applied to model different TIRM measuring schemes to find the most appropriate one for the determination of the distance between the particle and flat surface. It has been found that placing the collector beneath the prism gives a considerable advantage as compared to the conventional TIRM design.

INTRODUCTION

Total Internal Reflection Microscopy (TIRM) is an effective modern technique to measure weak interactions between colloidal particles in solution and a flat surface with high resolution up to 1nm [1]. The high sensitivity of TIRM is due to the use of the Brownian fluctuations of a free colloidal particle for obtaining the interaction potential. Another advantage of TIRM is that it is appropriate for measurements close to the surfaces. Recently, TIRM has been applied to measure van der Waals, Casimir, magnetic, depletion, and electrostatic forces [2].

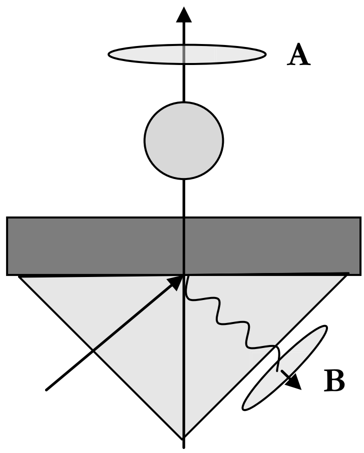

Figure 1. Model geometry: objective placement for the cases of conventional setup (A) and alternative setup (B).

In the beginning of TIRM measurements, the reconstruction of the interaction potential was based on the simple assumption that the intensity of the field scattered by a particle is

* Corresponding author: Natalia Grishina (ngrishina@inbox.ru)

proportional to the intensity of an evanescent exciting field in the particle domain. Later measurements showed deviation from the behaviour predicted by the simple model. Recently, the simple assumption has been corrected by using the rigorous scattering model based on the Discrete Sources Method (DSM) [3]. It has been proved that the use of the DSM model allows reconstruction of the potential with high precision [4].

In TIRM setups, the colloidal particle is situated above the glass prism. The laser beam propagating from the prism to the surface with an angle slightly above the angle of total internal reflection generates an evanescent field in the area above the prism. In conventional TIRM schemes the scattered light is collected by objective placed above the particle in the far zone (Fig. 1A). Recently, it has been suggested to place the light collector in the area beneath the glass prism. In this case, the collecting objective is oriented toward the beam which is reflected from the prism surface (Fig. 1B). To reduce the intensity of the reflected beam the effect of Plasmon Resonance (PR) in a thin gold film, deposited on the prism surface, can be employed [5]. In the present work, DSM has been applied to model and compare the efficiency of both of the suggested TIRM schemes. The direct comparison allows choosing the scheme that is more appropriate for the determination of the distance between particle and film.

Discrete Sources Method

In this section, we start with the mathematical statement of the polarized light scattering problem. Consider a glass prism occupying a half-space D_1, where $z < 0$, with a metal film of thickness *d* occupying D_f, where $d > z > 0$, on the prism. An axially symmetric penetrable particle with interior domain D_i and smooth boundary ∂D is deposited above the film in the domain D_0, where $z > d$. Introduce a Cartesian coordinate system *Oxyz* by choosing its origin *O* at the prism surface Σ_0 and the *z*-axis coincides with the axis of symmetry of the particle and is directed into the domain D_0. We assume that the exciting field $\{\mathbf{E}^0, \mathbf{H}^0\}$ is a linearly polarized plane wave propagating inside the glass prism at an angle θ_1 with respect to the *z*-axis. Then, the mathematical statement of the scattering problem includes:

- the Maxwell equations, in $D_{1,f,i,0}$;
- transmission conditions at the prism, film $\Sigma_{0,f}$, and particle ∂D surfaces;
- the radiation conditions in $D_{1,0}$ and attenuation condition in D_i at infinity.

The solution of the boundary value problem (BVP) is constructed following the DSM requirements [3]. First, the diffraction problem of the plane wave $\{\mathbf{E}^0, \mathbf{H}^0\}$ on the layered plane interface is solved. The resulting field $\{\mathbf{E}^0_\zeta, \mathbf{H}^0_\zeta\}, \zeta = 0, 1, f$ satisfies the transmission condition at $\Sigma_{0,f}$. Then, we construct an approximate solution of the BVP for the scattered field $\{\mathbf{E}^s_\zeta, \mathbf{H}^s_\zeta\}$ in the domains D_ζ, $\zeta = 0, f, 1$ and the total field inside the particle D_i. According to the DSM scheme, the electromagnetic fields are represented as a finite linear combination of multipole fields that satisfy analytically the following: the Maxwell equations in the domains $D_{0,1,f}$ as well as the infinity conditions and the transmission conditions at the plane interfaces $\Sigma_{0,f}$. Thus, the scattering problem is reduced to the problem of approximating the exciting field on the particle surface ∂D. The amplitudes of Discrete Sources (DS) are to be determined from the transmission conditions at the particle surface ∂D

$$\mathbf{n}\times(\mathbf{E}_i-\mathbf{E}_0^s)=\mathbf{n}\times\mathbf{E}_0^0,\ \mathbf{n}\times(\mathbf{H}_i-\mathbf{H}_0^s)=\mathbf{n}\times\mathbf{H}_0^0 \tag{1}$$

Here $\mathbf{n}$ is the normal to ∂D, and $\{\mathbf{E}_i,\mathbf{H}_i\}$ is the internal field.

To construct the fields of dipoles and multipoles that analytically satisfy the transmission conditions at the plane interfaces $\Sigma_{0,f}$, the Green's tensor for a stratified interface is used. Since the scattering problem geometry is axially symmetric with respect to the z-axis and the multipoles are distributed along the axis of symmetry, fulfilling the transmission conditions at the surface ∂D can be reduced to the sequential solution of the transmission problems for the Fourier harmonics of the fields involved in (1). So, instead of matching the fields on the scattering surface, we can match their Fourier harmonics, thus reducing the approximation problem on the surface to a set of problems enforced at the particle surface generatrix $\Im$. The one-dimensional problems are solved by the Generalized Point Matching Technique. In this approach, the Fourier harmonics of the DSM amplitudes are determined by solving over-determined linear systems and computing a normal pseudo-solution.

The approximate solution converges to the exact one since the systems of the fields of the dipoles and multipoles are complete. As DSM is a direct method, it allows solving the scattering problem for the entire set of incident angles θ_1 and both polarizations (*P* and *S*) at once. Besides, the numerical scheme provides an opportunity to control the convergence of the approximate solution by posterior error estimation [3].

After the amplitudes of the DSM are determined, one can compute the far field pattern of the scattered field in the far zone of $D_{1,0}$ as

$$\mathbf{E}_{0,1}^s(\mathbf{r})\big/\left|\mathbf{E}^0(0)\right|=\frac{\exp\{jk_{0,1}r\}}{r}\mathbf{F}_{0,1}(\theta,\varphi)+O\left(r^{-2}\right),\ r\to\infty \tag{2}$$

Here, $\mathbf{F}_{0,1}^{P,S}(\theta,\varphi)$ is a far field pattern at the unit sphere corresponding to P/S polarized excitation. It is represented by finite linear combinations of elementary functions. This circumstance ensures fast and effective computer analysis of the scattering characteristics in the far zone.

Presently, we consider the scattered intensity in $D_{0,1}$, $I_{0,1}^{P,S}(\theta_1,\theta,\varphi)=\left|\mathbf{F}_{0,1}^{P,S}(\theta_1,\theta,\varphi)\right|^2$, and the objective response $\sigma_{0,1}^{P,S}(\theta_1)=\int\limits_{\Omega_{0,1}} I_{0,1}^{P,S}(\theta_1,\theta,\varphi)d\omega$. Here the solid angles correspond to collectors deposited above or beneath the prism.

NUMERICAL RESULTS

Below some numerical results are presented. In Fig. 2, the objective response versus the particle-film distance is presented for the particle of diameter D=350 nm, for collectors deposited above the prism (conventional setup) and beneath the prism. From direct comparison, it is obvious that the interpretation of the measuring results for the standard scheme is complicated due to the non-monotonic behaviour of the curves for both polarizations. Unlike the conventional setup, the objective response of the scattered light observed beneath the prism behaves monotonically and should be easier to analyse. In Fig. 3, the intensity of the scattered light from the particle D=350 nm is presented in polar coordinates in the plane of the incident wave for two particle-film distances, which correspond to the positions of the local minima and maxima of the intensity curve for P polarization above the prism. These distances provide almost the same values of the collector's response for the conventional TIRM setup (see Fig. 2).

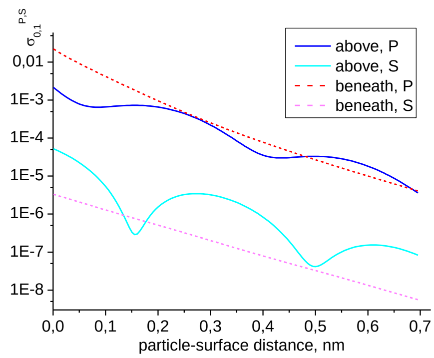

Figure 2. Objective response vs. the particle-film distance for the particle D=350 nm, collectors deposited above and beneath the prism. P and S polarizations.

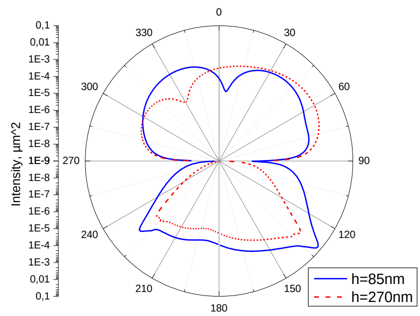

Figure 3. Intensity of the scattered light for the particle D=350 nm in the plane of incidence, two particle-film distances. P polarization.

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