Light scattering by horizontally oriented particles: Symmetry properties of the phase matrix

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Using elementary symmetry considerations ,we present seven symmetry relations for the phase matrix of horizontally oriented particles. These relations have a wide range of validity and hold for all directions of incident and scattered electromagnetic radiation.

INTRODUCTION

We consider light scattering by a small volume element in a medium containing independently scattering particles that are horizontally oriented. Examples of such particles are various types of hydrometeors, like snow flakes and ice crystals. Suppose the volume element is located at the origin of a coordinate system with axes x, y, and z (see Fig. 1). We call the x, y plane the horizontal plane and the positive z-axis the local vertical. The matrix transforming the Stokes parameters of the incident beam into those of the scattered beam is the phase matrix. Here the meridian planes of both beams are used as a plane of reference for the Stokes parameters.

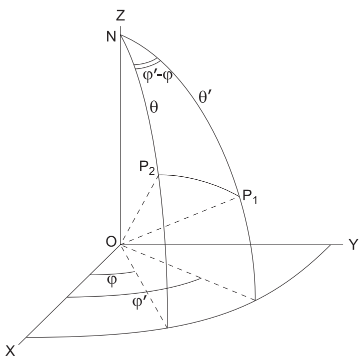

Figure 1. Scattering by a local volume-element at O. Points N, P_1 and P_2 are located on a unit sphere. The direction of the incident light is OP_1 and that of the scattered light is OP_2.

The phase matrix of a volume element plays a key role in studies of light scattering. It occurs, for instance, as the kernel in the equation of radiative transfer [1, 2, 3, 4, 5, 6, 7]. The phase matrix depends, in general, on the angles θ' and φ' of the incident beam and θ and φ of the scattered beam. The azimuthal angles are measured clockwise from the positive x-axis when looking in the direction of the upward vertical. If there is rotational symmetry

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about the local vertical the azimuth dependence reduces to the difference $\varphi - \varphi'$. Making the Stokes parameters I, Q, U and V of a beam of light the elements of a Stokes vector, **I**, the scattering process can be written as

$$\mathbf{I}^s(u,\varphi) = c\mathbf{Z}(u,u',\varphi-\varphi')\mathbf{I}^i(u',\varphi') \quad (1)$$

where the superscripts s and i refer to the scattered and incident beam, respectively, $u = -\cos\theta$ and $u' = -\cos\theta'$, c is a scalar that can be used for normalization purposes, and $\mathbf{Z}(u,u',\varphi-\varphi')$ is the phase matrix of the volume element.

Quite general symmetry relations for the phase matrix of collections of particles that are randomly oriented in three-dimensional space have been presented earlier [1]. The main purpose of this contribution is to investigate, by means of elementary symmetry arguments, which symmetry relations hold for the phase matrix of horizontally oriented particles.

We consider particles having a plane of symmetry. Such particles are identical to their mirror particles. As an example, we first consider collections of hexagonal plates with two broad flat sides in the horizontal plane, but with random orientation in that plane.

SYMMETRY RELATIONS

Reciprocity

The reciprocity principle corresponds to invariance of the ratio cause/effect under inversion of time. This amounts to changing u' into $-u$ and u into $-u'$, as well as interchanging φ and φ' (see Fig. 1). The result is the reciprocity relation

$$\mathbf{Z}(-u',-u,\varphi'-\varphi) = \mathbf{P}\tilde{\mathbf{Z}}(u,u',\varphi-\varphi')\mathbf{P}, \quad (2)$$

where **P** is the 4×4 diagonal matrix (1,1,-1,1) and a tilde above a matrix stands for the transposed matrix.

Eq. (2) is well known from the theory for scattering by randomly oriented particles [1, 3]. Since reciprocity holds, under certain conditions, for each individual particle in arbitrary orientation the validity of Eq. (2) does not depend on the orientation of the particles [8, 9]. Sufficient conditions for reciprocity are that the dielectric, permeability and conductivity tensors of the particles are symmetric and magnetic fields can be ignored. We shall henceforth assume that reciprocity holds for all particles considered in this work.

Mirror symmetry

It is clear that we have mirror symmetry with respect to the meridian plane of incidence. Referring to Fig. 2, it is readily seen that, if an incident beam i_1 gives rise (among others) to a scattered beam r_1, then the incident beam i_2, which is the mirror image of i_1 with respect to the meridian plane of incidence, gives rise (among others) to the beam of scattered light r_2, which is the mirror image of r_1 with respect to the meridian plane of incidence. Now i_1 and i_2 differ in the signs of their third and fourth Stokes parameters and so do r_1 and r_2. Furthermore, we have $\varphi_2 - \varphi_0 = \varphi_0 - \varphi_1$. Using Eq. (1) first for r_1 and then for r_2, we find the following mirror symmetry relation for the phase matrix

$$\mathbf{Z}(u, u', \varphi' - \varphi) = \mathbf{PQZ}(u, u', \varphi - \varphi')\mathbf{QP}, \tag{3}$$

where **Q** is the 4×4 matrix diag (1,1,1,-1), so that **PQ** = **QP** = diag (1,1,-1,-1). An interesting corollary of Eq. (3) is that, if a Fourier-series expansion is used to handle the azimuth dependence of the phase matrix, the 2×2 submatrices in the upper left corner and the lower right corner contain only cosine terms and in general an azimuth independent term, whereas the other two 2×2 submatrices possess only sine terms or vanish.

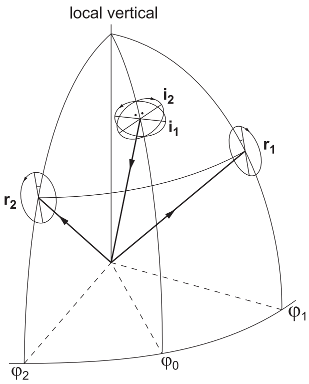

Figure 2. Illustration of the mirror symmetry relation for the phase matrix. If the incident beam i_1 gives rise (among others) to the beam of scattered light r_1, then the incident beam i_2, which is the mirror image of i_1 with respect to the plane of incidence, gives rise (among others) to the beam of scattered light r_2, which is the mirror image of r_1 with respect to the plane of incidence. The position angles of the polarization ellipses of the incident light (dots) and scattered light (small arcs) are also indicated, as well as the sense in which the four polarization ellipses are traced. [After Hovenier, [10]]

Rotational symmetry

We have assumed rotational symmetry about the vertical (azimuthal symmetry). Therefore, simultaneous rotation of the meridian planes of incidence and scattering about the vertical, through any angle, gives no new symmetry relation. However, rotation of the horizontal plane, together with the directions of the incident and scattered light, about a horizontal axis over an angle π gives physically the same scattering problem, but the sign of the azimuth difference must be switched [1]. This yields the symmetry relation

$$\mathbf{Z}(-u, -u', \varphi' - \varphi) = \mathbf{Z}(u, u', \varphi - \varphi'). \tag{4}$$

Combinations

By combining the three fundamental symmetry equations Eq. (2), Eq. (3), and Eq. (4) we find

$$\mathbf{Z}(-u', -u, \varphi - \varphi') = \mathbf{Q}\tilde{\mathbf{Z}}(u, u', \varphi - \varphi')\mathbf{Q}. \tag{5}$$

$$\mathbf{Z}(u', u, \varphi - \varphi') = \mathbf{P}\tilde{\mathbf{Z}}(u, u', \varphi - \varphi')\mathbf{P}. \tag{6}$$

$$\mathbf{Z}(-u, -u', \varphi - \varphi') = \mathbf{PQZ}(u, u', \varphi - \varphi')\mathbf{QP}. \tag{7}$$

and

$$\mathbf{Z}(u', u, \varphi' - \varphi) = \mathbf{Q}\tilde{\mathbf{Z}}(u, u', \varphi - \varphi')\mathbf{Q}. \tag{8}$$

CONCLUSIONS

Using symmetry arguments, we have found seven symmetry relations for the phase matrix of horizontally oriented hexagonal plates. It is, however, directly clear from the symmetry arguments that these relations must hold for many other kinds of horizontally oriented particles. These include hexagonal columns with their long axes randomly oriented in a horizontal plane and randomly rotated about their long axes. As long as the particles and their orientation distribution are such that the three fundamental symmetry relations Eq. (2), Eq. (3), and Eq. (4) hold, all seven symmetry relations are valid. A more extensive treatment of the topic of this abstract is given in [11].

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