Diffraction of a plane wave on a grating consisting of impedance bodies of revolution

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The three-dimensional problem of plane electromagnetic wave scattering on a grating consisting of coaxial impedance bodies of revolution is considered. An infinite system of integral equations, to which the initial problem is reduced, is derived. An efficient algorithm for the calculation of the periodic Green's function is offered. The angular dependence of the scattering pattern is obtained.

INTRODUCTION

The paper considers diffraction of a plane electromagnetic wave on an infinite periodic grating consisting of impedance bodies of revolution located at one axis. To solve the problem, we use a modified null field method (MNFM), which has previously been successfully applied in [1, 2]. The null field method (NFM), often named in the literature also as a method of T-matrix [3], has been offered for the first time by Waterman [4]. The basis for the method is a certain relation (see below) which is satisfied everywhere inside the scatterer. If we require that this relation is fulfilled on some closed surface inside the scatterer, the initial boundary problem is reduced to the integral equation of the first kind relative to an unknown current distributed on the surface of the body. In [1, 2], it has been shown that the integral equation has the solution corresponding to the boundary problem, if and only if the surface (designated in these works with the letter Σ) on which the condition of the null field is fulfilled, covers the set of singularities of analytical continuation of the diffracted field inside the scatterer. Besides, it is shown that, for the development of high-speed and stable algorithms, the surface Σ should be constructed by means of analytical deformation of the surface of the scatterer [5].

Notice that, in solving the considered problem, we face the development of an efficient algorithm for the calculation of the periodic Green's function. We calculate the Green's function by the method analogous to the approach proposed in [6], which considered the problem of diffraction on a body in a circular waveguide.

DERIVATION OF THE MAIN RELATIONS

Consider a grating consisting of identical coaxial impedance bodies of revolution. We assume that the grating has a period d . Introduce a Cartesian coordinate system and direct the z -

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axis along the axis of the grating. Denote by S_0 the surface of the central element of the grating. We suppose that the structure is irradiated by the plane wave:

$$\vec{E}^0 = \vec{p}_0 \exp(-ikr(\sin\theta_0 \sin\theta \cos(\varphi - \varphi_0) + \cos\theta_0 \cos\theta)), \tag{1}$$

where (r, θ, φ) are the spherical coordinates, k is the wave number, and θ_0, φ_0 are the incidence angles of the plane wave. The diffracted field outside the grating obeys the homogeneous Maxwell equations and also satisfies the Floquet periodic conditions:

$$\vec{E}^1(\rho, \varphi, z + d) = \vec{E}^1(\rho, \varphi, z)\exp(-i\kappa), \tag{2}$$

where $\kappa = kd\cos\theta_0$ is the Floquet parameter and (ρ, φ, z) are the cylindrical coordinates. The formulas for the magnetic field are similar. The diffracted field also obeys the radiation condition at infinity. On the surface of each element of the grating, the impedance boundary condition

$$\vec{n} \times \vec{E} = Z_0\, \vec{n} \times (\vec{n} \times \vec{H}) \tag{3}$$

is satisfied. Here $\vec{n}$ is the outward normal and Z_0 is the impedance.

Let us apply MNFM. For this aim, we construct the auxiliary surface Σ_0 which is located inside the original surface S_0 of the central element of the grating. If the equation of the surface S_0 in the spherical coordinate system has the form $r = r(\theta)$, the auxiliary surface is defined by the equations: $x = r_\Sigma \sin\theta_\Sigma \cos\varphi, \quad y = r_\Sigma \sin\theta_\Sigma \sin\varphi, \quad z = r_\Sigma \cos\theta_\Sigma$, where [5]

$$\theta_\Sigma = \arg\xi(\tau), \quad r_\Sigma = |\xi(\tau)|, \quad \xi(\tau) = r(\tau + i\delta)\exp(i\tau - \delta), \tag{4}$$

In formulas (4) δ is a positive parameter responsible for the degree of deformation of the contour of the body axial cross section and $\tau \in [0, \pi]$. The choice of the parameter δ is detailed in [1, 2, 5]. In accordance with MNFM we state the following condition at the auxiliary surface Σ_0:

$$\vec{n} \times \int_{S_0} \left[-i\varsigma\left(-(\vec{J}, \nabla' G)\nabla G + k^2 \vec{J} G\right) + kZ_0 \left(\nabla G \times (\vec{n}' \times \vec{J})\right)\right] ds' = -\vec{n} \times \vec{E}^0, \tag{5}$$

where $\vec{r} \in \Sigma_0$, $\vec{J}$ is the unknown current on the surface S_0 of the central element of the grating, and ς is the wave impedance. This equation is solvable only on condition that the surface Σ_0 covers the set of the singularities of the analytical continuation of the diffracted field inside S_0. The function G in Eq. (5) is the periodic Green's function:

$G(\vec{r},\vec{r}') = \sum_{s=-\infty}^{\infty} G_0(R_s)\exp(-is\kappa)$, where

$$G_0(R_s) = \frac{\exp(-ikR_s)}{4\pi\, kR_s}, \; R_s = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos\psi + (z - z' - sd)^2}, \; \psi = \varphi - \varphi'. \quad (6)$$

Expand the unknown current $\vec{J}$ and the Green's function into the Fourier series:

$$\vec{J}(t,\varphi) = \sum_{n=-\infty}^{\infty} \vec{I}_n(t)\exp(in\varphi), \quad (7)$$

$$G(r,\theta,r',\theta',\varphi-\varphi') = \sum_{m=-\infty}^{\infty} S_m(r,\theta,r',\theta')\exp(im(\varphi-\varphi')). \quad (8)$$

Then we define $\vec{I}_m(t) = I_m^1(t)\frac{r'(t)}{r(t)}\vec{i}_r + I_m^1(t)\vec{i}_\theta + I_m^2(t)\vec{i}_\varphi$, where $\vec{i}_r$, $\vec{i}_\theta$, $\vec{i}_\varphi$ the unit vectors of the spherical coordinates and the prime denotes the derivative by the corresponding argument. Using formulas (5) – (8), one can obtain the following system of integral equations:

$$\begin{cases} \int_0^\pi K_m^{11}(\tau,t)I_m^1(t)dt + \int_0^\pi K_m^{12}(\tau,t)I_m^2(t)dt = B_m^1(\tau), \\ \int_0^\pi K_m^{21}(\tau,t)I_m^1(t)dt + \int_0^\pi K_m^{22}(\tau,t)I_m^2(t)dt = B_m^2(\tau), \end{cases} \quad (9)$$

where $m = 0, \pm1, \pm2, \ldots,\quad \tau \in [0,\pi]$. The kernels of this system are expressed by the coefficients S_m and their derivatives. Note that the system (9) is solved by the collocation technique [1, 2, 5].

NUMERICAL RESULTS

To test the method, we consider the problem of wave scattering by a grating consisting of closely-spaced superellipsoids of revolution. The axial cross-section of the superellipsoid is defined by the equation

$$\left(\frac{x}{a}\right)^{2l} + \left(\frac{z}{c}\right)^{2l} = 1.$$

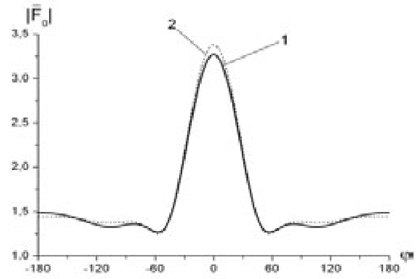

Figure 1.

For large values of the parameter l and small distances between the scatterers, the problem of such a geometry is only slightly different from the two-dimensional problem of scattering by an infinite circular cylinder (it is assumed that the plane wave is incident perpendicular to the axis z and the electric vector is parallel to the axis of the structure). As is well known, this problem has an analytical solution. In Fig. 1, the distribution of the pattern $\vec{F}_0(\theta_0, \varphi)$ of zero mode of the grating is presented. In cylindrical coordinates, the pattern is defined by the formula

$$\vec{E}^1 \simeq -\frac{i\pi}{2kd}\sqrt{\frac{2}{\pi}}e^{i\pi/4}\sum_{s=-\infty}^{\infty}\vec{F}_0(\theta_s,\varphi)\frac{\exp(-iv_s\rho - iw_s z)}{\sqrt{v_s\rho}}, \tag{10}$$

where $w_s = (\kappa + 2\pi s)/d$, $v_s = \sqrt{k^2 - w_s^2}$, $\theta_s = \arccos(w_s)$. The sign of square root is chosen so that its imaginary part is not positive. The parameters of the problem are the following: $ka = 2.5;$ $kc = 5;$ $l = 10$, period of the grating $kd = 10.1$, $\varphi_0 = 0$, $\theta_0 = \pi/2$, $\vec{p}_0 = \vec{i}_z$. Curve 1 in Fig. 1 demonstrates the dependence of the pattern for the grating consisting of the superellipsoids and curve 2 corresponds to the case of scattering by the infinite circular cylinder with radius $ka = 2.5$. One can see rather small differences between the dependences.

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