Scattering of light by Gaussian-random-ellipsoid particles

K. Muinonen*,[1,2] and T. Pieniluoma[1]

[1]*Department of Physics, P.O. box 64, FI-00014 University of Helsinki, Finland.*
[2]*Finnish Geodetic Institute, P.O. box 15, FI-02431 Masala, Finland.*

We introduce the stochastic geometry of a Gaussian random ellipsoid (GE) and, with the discrete-dipole approximation, carry out preliminary computations for light scattering by wavelength-scale GE particles. We compare the scattering characteristics of GE particles to those of perfect ellipsoids.

INTRODUCTION

Natural small particles may exhibit irregular shapes with preferential elongation or flattening. Here the shapes of such irregular small particles are modeled using the stochastic geometry of what we call a Gaussian random ellipsoid (GE). GE is a natural extension for the Gaussian random sphere (GS; e.g. [1, 2]) and GE transforms to GS in the limit of vanishing elongation and flattening.

Scattering properties for GE particles are studied here with the discrete-dipole approximation. DDA is a flexible method for numerical solution of scattering by irregular particles (e.g. [3]). We utilize the Amsterdam DDA code by Yurkin et al. [4]. In what follows, we introduce the stochastic geometry for GE. We then proceed to present the first DDA computations for scattering by GE particles.

GAUSSIAN RANDOM ELLIPSOID

In GE, lognormal height statistics are imposed on a base ellipsoid along the local normal direction. As compared to GS, GE introduces two additional shape parameters: the axial ratio $b : a$ or, equivalently, the elongation $(a - b) : a$; and $c : b$ or the flattening $(b - c) : b$.

The ellipsoidal base geometry raises fundamental issues concerning the homogeneity of the superimposed statistics. In GS, the great-circle distance utilized in the correlation of two radial distances can be interpreted in two ways: first, the distance can be taken literally as the great-circle angle between the two points; second, it can be unambiguously mapped to the Cartesian distance for the two points on the base sphere. In a corresponding way for GE, the distance between two points on the base ellipsoid can be measured along the geodetic line connecting the points or as the Cartesian distance between the points. In the present context, we utilize the Cartesian distance in correlating heights on the base ellipsoid.

Due to the requirement of height variation along the local normal vector, further constraints must be introduced for the mean height corresponding to the mean radial distance in GS. We define the mean height h to coincide with the minimum radius of curvature for the base ellipsoid with semiaxes a, b, and c. This implies that the single center point of GS

*Corresponding author: Karri Muinonen (karri.muinonen@helsinki.fi)

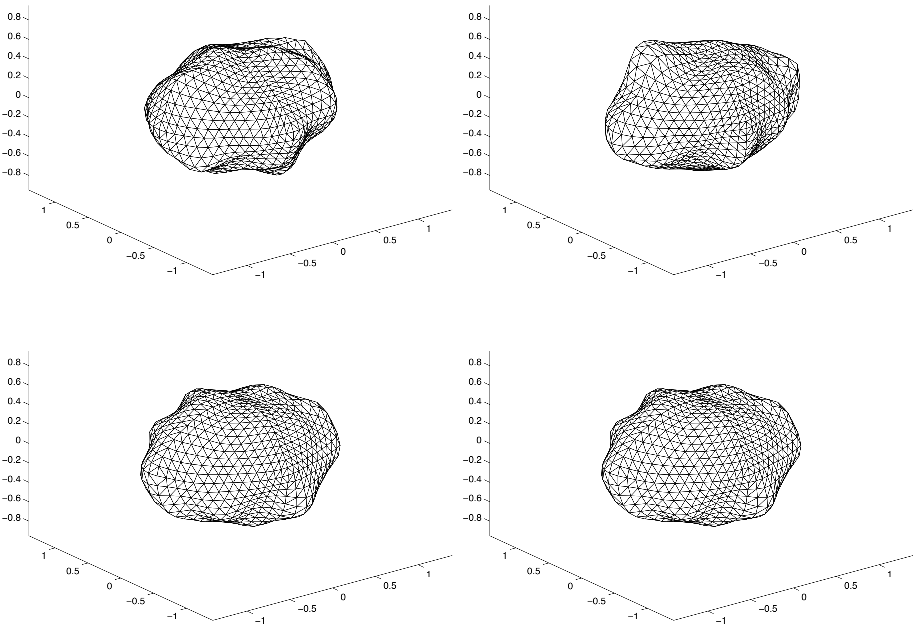

Figure 1. Sample Gaussian ellipsoids with $a : b : c = 1 : 0.7 : 0.6$, $\sigma = 0.05$, and $\ell = 0.2$.

evolves into a surface of center points for GE (note that this surface is not ellipsoidal in shape). In summary, the position of any point on a sample GE can be expressed as

$$\boldsymbol{r}(\vartheta, \phi) \quad = \quad \boldsymbol{r}_{\mathrm{E}}(\vartheta, \phi) + h\left[\exp\left(s(\vartheta, \phi) - \frac{1}{2}\beta^2\right) - 1\right]\boldsymbol{n}(\vartheta, \phi), \qquad (1)$$

where ϑ, ϕ are the polar and azimuthal angles of the spherical coordinate system, $\boldsymbol{r}_{\mathrm{E}}(\vartheta, \phi)$ and $\boldsymbol{n}(\vartheta, \phi)$ denote the local position and unit outward normal vectors on the base ellipsoid, respectively, s is the logarithmic height and a Gaussian random variable, and β is the standard deviation of s. The relative variance of heights is $\sigma^2 = \exp(\beta^2) - 1$. Note, in particular, that $\boldsymbol{r}(\vartheta, \phi)$ no longer points in the direction specified by the spherical coordinates ϑ, ϕ.

RESULTS AND DISCUSSION

We compute scattering matrices for GE particles with size parameters $ka = 3$ or $ka = 6$, complex refractive index $m = 1.55 + \mathrm{i}0.001$, standard deviation $\sigma = 0.05$, correlation length $\ell = 0.2$ in a Gaussian correlation function $C_s(d) = \exp(-\frac{1}{2}\frac{d^2}{\ell^2})$ (d is the Cartesian distance between two points on the base ellipsoid), and axis ratio $a : b : c = 1 : 0.7 : 0.6$ (see Fig. 1 for sample shapes). The scattering characteristics are ensemble-averaged over 100 GE realizations and the scattering characteristics are orientation averaged over 242 different orientations for each realization. The scattering volume is discretized into $32\times 32\times 32 =$

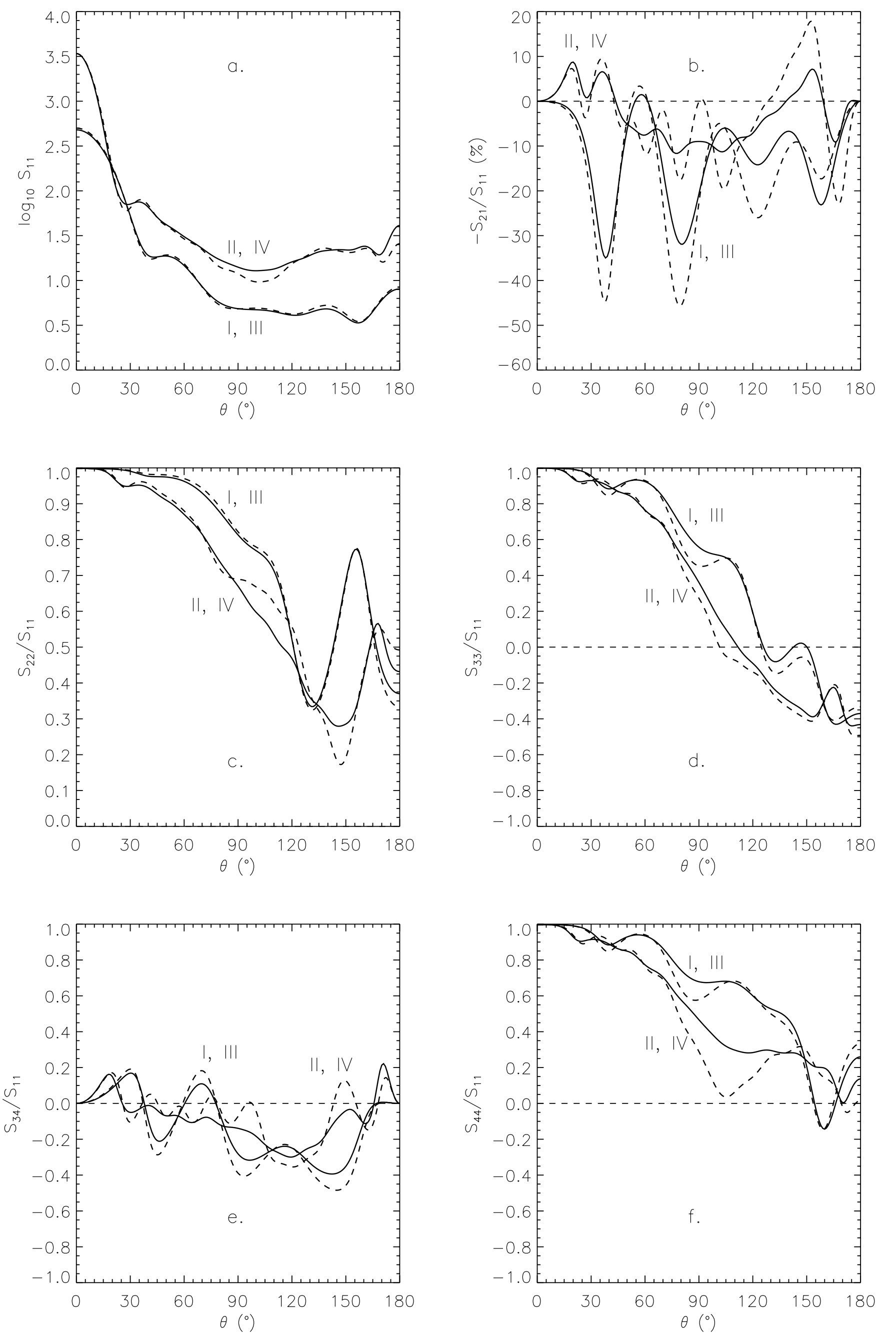

Figure 2. Ensemble-averaged angular scattering characteristics for GE particles with base-ellipsoid axial ratio $a : b : c = 1 : 0.7 : 0.6$ and complex refractive index $m = 1.55+\mathrm{i}0.001$ using Discrete-Dipole Approximation: a. Scattering-matrix element S_{11}; b. Degree of linear polarization $-S_{21}/S_{11}$; c. S_{22}/S_{11}; d. S_{33}/S_{11}; e. S_{34}/S_{11}; f. S_{44}/S_{11}. Four cases are shown: I. GE with $ka = 3$ (solid line); II. GE with $ka = 6$ (solid line); III. base ellipsoid with $ka = 3$ (dashed line); IV. base ellipsoid with $ka = 6$ (dashed line).

32768 cubic cells for $ka = 3$ or into $64 \times 64 \times 64 = 262144$ cells for $ka = 6$, well within the validity criteria of DDA.

Fig. 2 illustrates the results of the scattering computations. The GE particles exhibit overall angular characteristics commonly encountered in scattering experiments for small particles as well as numerical computations for other irregular particles. The scattering-matrix element S_{11} shows the precursor of the forward diffraction pattern as well as increased backward scattering. The increased backward scattering can also be envisaged as a deep minimum next to the backward scattering direction caused by a destructive interference (e.g., [5]). The degree of linear polarization $-S_{21}/S_{11}$ shows clear negative polarization at intermediate scattering angles (cf. [6]) as well as pronounced branches of negative polarization near backscattering. The remaining scattering patterns S_{22}/S_{11}, S_{33}/S_{11}, S_{34}/S_{11}, and S_{44}/S_{11} also resemble those obtained for GS particles.

Comparison to scattering by regular base ellipsoids shows that adding irregularity on the base ellipsoid results in smoothing of all angular patterns. The scattering-matrix elements S_{11} are quite similar for randomly oriented base ellipsoids and Gaussian ellipsoids. Of all the angular patterns, the degree of linear polarization $-S_{21}/S_{11}$ appears to be most sensitive to the surface irregularities.

CONCLUSION

In the future, we will develop a Gaussian random ellipsoid where the correlation is measured along the geodetic line on the ellipsoid. In addition, the Gaussian-ellipsoid geometry can turn useful in physical studies of small solar-system bodies.

The direct problem of light scattering involves the computation of scattering by small particles with varying size, shape, and refractive index or optical properties in general. The inverse problem concerns retrieving particle properties based on observations or laboratory measurements of their scattering and absorption properties. We envisage that the Gaussian random ellipsoid can become a useful tool for irregular small particles in inverse problems.

REFERENCES

[1] K. Muinonen, T. Nousiainen, P. Fast, K. Lumme, and J.I. Peltoniemi. Light scattering by Gaussian random particles: ray optics approximation. JQSRT **55** (1996).

[2] J.I. Peltoniemi, K. Lumme, K. Muinonen, and W.M. Irvine. Scattering of light by stochastically rough particles. Appl. Opt. **28** (1989).

[3] E.S. Zubko, Yu.G. Shkuratov, M. Hart, J. Eversole, and G. Videen, Backscattering and negative polarization of agglomerate particles. Opt. Lett. **28**(17) (2003).

[4] M.A. Yurkin and A.G. Hoekstra. The discrete dipole approximation: an overview and recent developments. JQSRT **106** (2007).

[5] K. Muinonen, J. Tyynelä, E. Zubko, and G. Videen. Coherent backscattering in planetary regoliths. Light Scattering Reviews (in press) (2010).

[6] J. Tyynelä, E. Zubko, K. Muinonen, and G. Videen. Interpretation of single-particle negative polarization at intermediate scattering angles. This volume (2010).