Light scattering by large faceted particles

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The problem of calculation of the scattering matrices for large faceted particles within the framework of physical optics is split into three steps. First, we find the matrix in the geometric optics approximation. Second, we calculate the so-called Fraunhofer diffraction matrix. And, finally, the matrix desired is found as an integral transform of the geometric optics matrix by means of the Fraunhofer matrix.

INTRODUCTION

The problem of light scattering by large, as compared with the incident wavelength λ, non-spherical particles, i.e. $a >> \lambda$ where a is a characteristic particle size, is successfully solved on the basis of the Maxwell equations if the size parameter $\rho = a/\lambda$ does not exceed, the value of about 10 (see e.g. [1]). The opposite case $\rho >>1$ is conventionally calculated by use of geometric optics, in particular, by means of the ray-tracing technique [2]. To fill the gap of the intermediate case of $\rho \approx 10$, physical optics is an obvious and rather simple instrument [3-6]. However the physical-optics approximations are often restricted by taking into account only diffraction near the forward-scattering direction, which is produced by particle projections or shadows (e.g., [7]).

In the case of large faceted particles like atmospheric ice crystals, the problem of light scattering within the framework of physical optics can be analytically reduced to a simple extension of the geometric-optics solution that is the topic of this presentation.

GEOMETRIC-OPTICS AND PHYSICAL-OPTICS SCATTERED WAVES

In general, a solution for any electromagnetic wave scattered by a particle is defined by a superposition of the incident $\mathbf{E}^0$ and scattered $\mathbf{E}^s$ waves

$$\mathbf{E} = \mathbf{E}^0 + \mathbf{E}^s \quad (1)$$

at any spatial point $\mathbf{r}$. At large distances from the particle, the scattered wave $\mathbf{E}^s$ is conventionally considered on the scattering direction sphere $\mathbf{n}$ ($|\mathbf{n}| = 1$) as the function $\mathbf{E}^s(\mathbf{n})$. In

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the near zone, i.e. at a distance $r << ka^2$ where $k = 2\pi/\lambda$ is the wavenumber, the scattered wave for a large faceted particle becomes strictly a superposition of discrete plane-parallel beams leaving the particle at various propagation directions

$$\mathbf{E}^s = \sum_{j=0}^{\infty} \mathbf{E}_j^s \,. \tag{2}$$

Here every beam $\mathbf{E}_j^s(\mathbf{r})$ is characterized by its transversal size and shape, polarization and propagation direction $\mathbf{n}_j$. At large distance from the particle $r >> a$, the beam propagation directions $\mathbf{n}_j$ become equivalent to the scattering directions, and the scattered wave on the scattering direction sphere $\mathbf{n}$ is reduced to a singular function. This singular function is strictly the superpositions of the Dirac delta-functions $\delta(\mathbf{n} - \mathbf{n}_j)$. Fig. 1 shows an example of the functions calculated for a fixed particle orientation. Note that the superposition of Eq. (2) includes not only the beams produced by an arbitrary number of reflections and refractions on the particle facets but also a shadow-forming beam (see e.g. [6, 8]). Appearance of the shadow-forming beam follows immediately from the superposition of Eq. (1) by subtraction of the incident wave from the total one. The shadow-forming beam propagates in the incident direction $\mathbf{n}_0$ and its transversal shape corresponds to the particle projection.

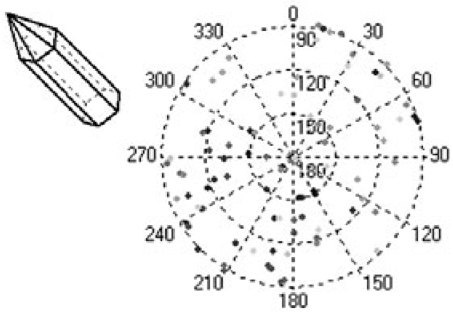

Figure 1. Phase function for a bullet ice crystal of a fixed orientation. The brightness of the dots is proportional to energy of the beams.

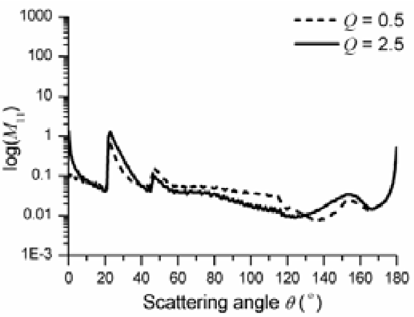

Figure 2. Phase functions for randomly oriented hexagonal ice plate and column (Q=diameter/length).

When the singular functions like those shown in Fig. 1 are averaged over particle orientations the δ-function singularities are smoothed in the majority of directions $\mathbf{n}$ resulting in some regular functions as shown in Fig. 2. Nevertheless some singularities still are left at certain directions manifesting themselves in Fig. 2 as sharp peaks. Note that the appearance of singularities is typical for the majority of functions obtained within the framework of geometric optics.

The superpositions of Eqs. (1) and (2) remain valid at arbitrary distances from the particle. As a result, in the wave zone $r >> ka^2$ every dot in Fig. 1 should be smeared into a Fraunhofer diffraction spot. In other words, the δ-functions $\delta(\mathbf{n} - \mathbf{n}_j)$ should be replaced by the Fraunhofer diffraction functions $F_j(\mathbf{n}-\mathbf{n}_j)$ obeying the same normalization $\int F_j(\mathbf{n})d\mathbf{n}=1.$ Similar replacements should also take place for any values averaged over an arbitrary probability distribution of particle orientations like the curves in Fig. 2. In general, we find the Mueller scattering matrices of the physical optics approach $\mathbf{M}_P$ become the following integral transform of the geometric optics matrix $\mathbf{M}_G$

$$\mathbf{M}_P(\mathbf{n},\mathbf{n}_0)=\int \mathbf{F}(\mathbf{n},\mathbf{n}')\mathbf{M}_G(\mathbf{n}',\mathbf{n}_0)d\mathbf{n}', \tag{3}$$

where the matrix **F** takes into account the Fraunhofer diffraction of the near-zone plane-parallel beams along with their interference on the scattering direction sphere **n** .

SPECULAR (ONCE-REFLECTED) SCATTERING

For a large faceted particle with random orientation, the light once–reflected by its facets can be calculated analytically. In our recent paper [9], we have considered the general case of an arbitrary distribution of such a particle over its orientation. In particular, if the normal **N** to a particle facet is distributed as p (**N**), the average specular intensity $I_G(\mathbf{n})$ produced by reflection from this facet is described by the following equation

$$I_G(\mathbf{n})=(s/4)R\left(\mathbf{n}\cdot\frac{\mathbf{n}-\mathbf{n}_0}{|\mathbf{n}-\mathbf{n}_0|}\right)p\left(\frac{\mathbf{n}-\mathbf{n}_0}{|\mathbf{n}-\mathbf{n}_0|}\right), \tag{4}$$

where s is the facet area and R is the Fresnel reflection coefficient.

For convex particles, the total specular scattering intensity is just a sum of intensities obtained by means of Eq. (4) for all facets. Then, if a facet can be approximately replaced by a circle, there is an analytical equation for the Fraunhofer diffraction functions (see Eq. (16) of Ref. [9]). And, finally, 2D-convolution of the geometric optics value of Eq. (4) with the diffraction function **F** results in the specular phase function in the physical optics approximation. This result is a simplified case of the general equation (3).

BACKSCATTERING BY HEXAGONAL ICE CRYSTALS

The proper geometric optics values for the case of ice hexagonal crystals were studied in details in our previous paper [10]. In particular, we proved that there are four types of photon trajectories that make contributions to backscattering. Unlike the abovementioned problem of specular scattering, this calculation is more complicated since we have to take into account the singularity appearing in the geometric optics approximation at the strict backward direction. We overcome these difficulties by cumbersome numerical calculations. The results obtained will be presented at the conference.

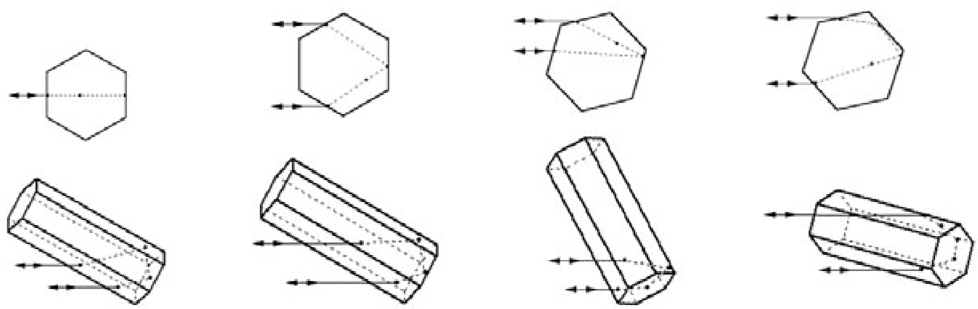

Figure 3. Four types of photon trajectories making main contribution to backscattering.

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