Asteroid spin and shape inversion for simulated Gaia photometry

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We present Markov-chain Monte-Carlo methods (MCMC) for the inversion of asteroid spins and shapes in the case of limited numbers of or sparsely distributed observations. We focus on convex optimization of asteroid spin and shape, obtaining realistic shape solutions, and exploring the regime of possible spin solutions. The asteroid shape is modeled as a triangulated surface with or without smoothing using bicubic splines. We apply the methods to simulated photometric data for the Gaia mission. We compare the inversion results obtained to the spin and shape originally used to generate the simulated photometric data.

INTRODUCTION

Gaia is an astrometric space mission of the European Space Agency (ESA), and a successor to the ESA Hipparcos mission. While determining and cataloging astrometric positions and movements of about one billion stars, Gaia will observe some hundreds of thousands of asteroids [1]. Photometric data of asteroids will consist of single brightness values ranging over a time interval of five years. This results in a maximum of about one hundred brightness values at varying observing geometries.

Here we apply Markov-chain Monte-Carlo methods [2] to simulated asteroid data in order to obtain spins and shapes of the simulated asteroids (for conventional convex inverse methods, see [3] and [4]). The inverted and original shape and spin solutions are compared to validate the applicability of the methods to the Gaia data.

MCMC METHODS

We have developed three inverse methods for three different purposes:

- In initial convex mapping, we explore the phase space using multiple chains in an optimization mode, accepting gradually improving solutions;
- In convex optimization, we start from the best solution from initial mapping, and gradually move toward the best-fit solution.
- In MCMC convex inversion, we explore the phase space in the neighborhood of the best-fit solution.

We make use of general convex shapes described using a large but finite number of triangles with or without smoothing using bicubic splines. In MCMC convex inversion, the

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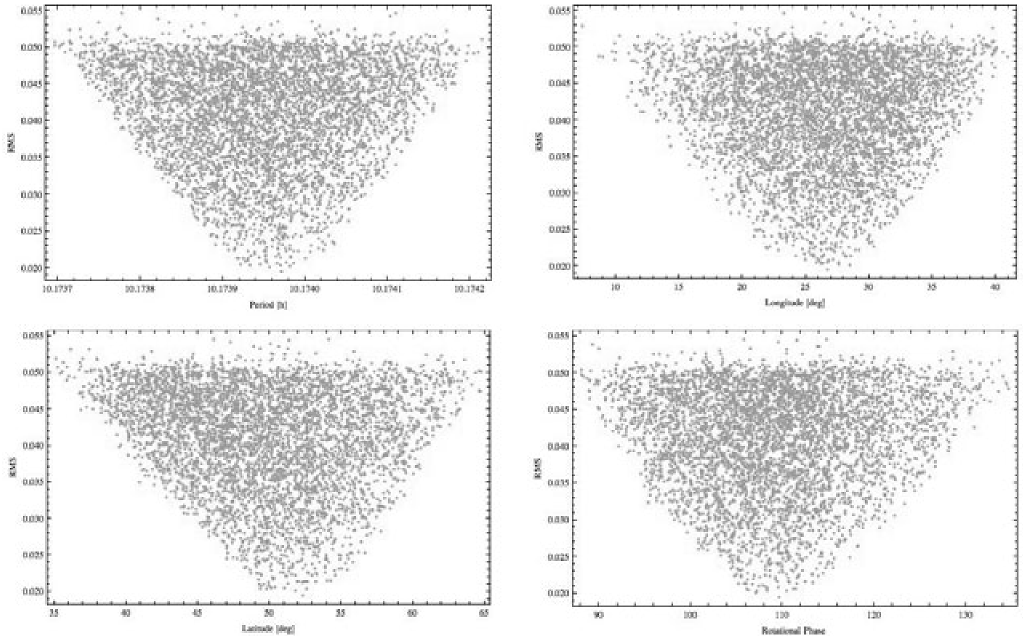
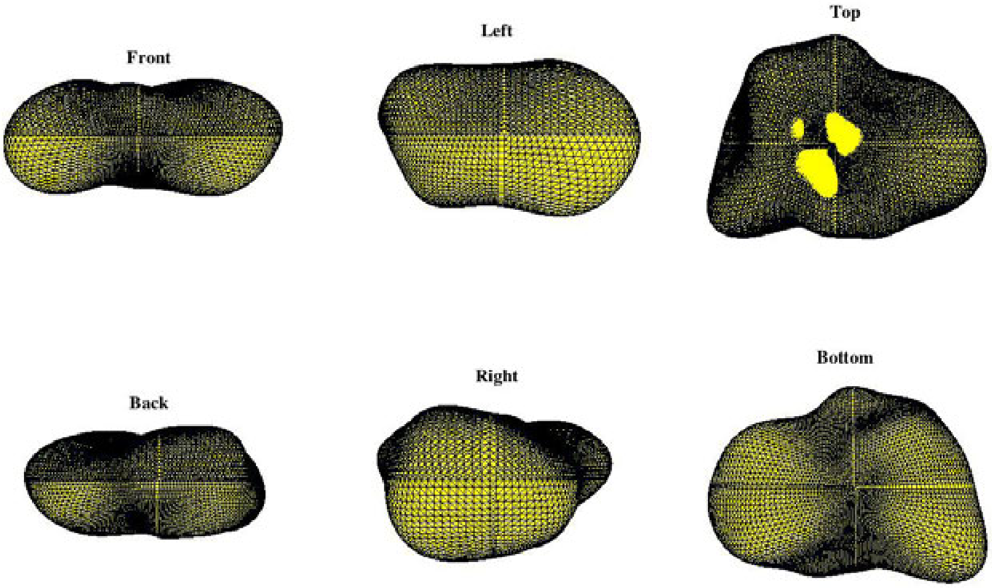


Figure 1. Rotational period and pole distributions as obtained from MCMC convex inversion. Original shape parameters: rotation period 10.17395622 (h), ecliptic longitude of rotational pole 25.02 (°), ecliptic latitude of rotational pole 62.89 (°), and rotational phase 110.1 (°).

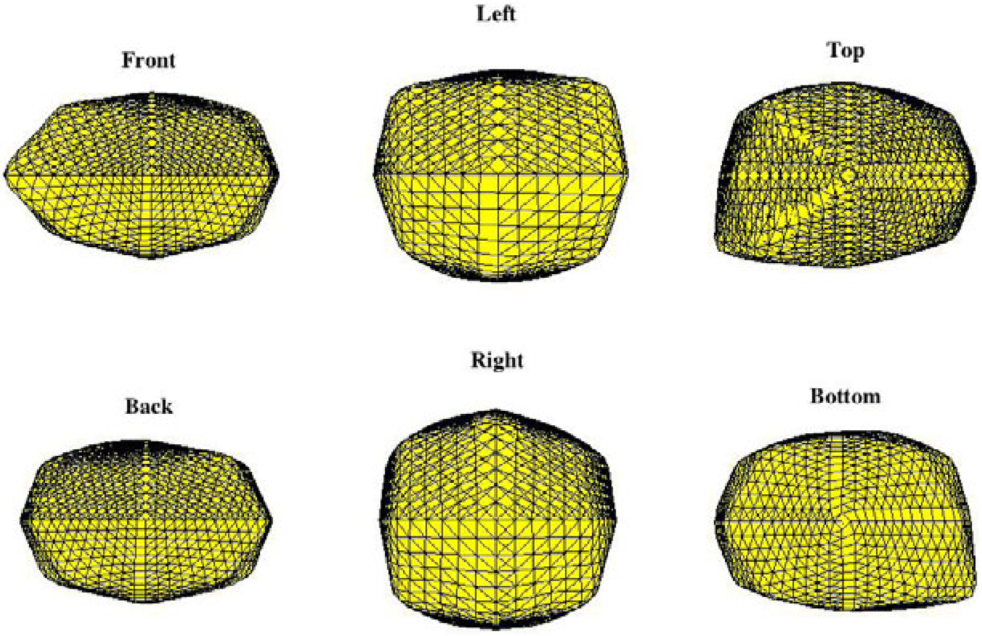
convex shape solutions are directly sampled. There are four parameters for the spin characteristics: the rotational period, the ecliptic longitude and latitude of the rotational pole, and the rotational phase of the object at a given time. The $3N$ Cartesian coordinates of the N triangle nodes constitute the free shape parameters. Altogether, there are $3 + 3N$ free parameters [2]. In MCMC convex inversion, the inversion parameters are sampled according to the Metropolis-Hastings algorithm [5]. The accepted shapes and spins generate a sequence, a Markov chain. The proposed spin and shape parameters are accepted or rejected depending on the a posteriori probability density values corresponding to the proposed and current parameters. If the proposed parameters provide a better fit to the data than the current, they are always accepted. If the fit is worse, the candidate is accepted with a certain probability.

RESULTS

We have applied the three inversion methods to the simulated Gaia data. The data was generated for a Gaussian sample sphere mimicking an asteroid. The observing geometries were those simulated for asteroid Vesta for the five-year mission duration [6]. The data for Vesta amounts to 69 brightness values. The accuracy of the simulated data was 0.01 mag. The possible shapes, the rotational period, the ecliptic longitude and latitude of the rotational pole are presented in Figs. 1 and 2. In order to validate the inversion methods, we compare



(a) Original shape



(b) Inverted shape

Figure 2. Original shape together with the one obtained with convex optimization. The best fit resulted in an rms-value of 0.02. In inversion, three triangle rows were utilized per octant.

the results to the original shape and spin values. The distributions for the pole orientation in Fig. 1 and shape in Fig. 2 are in agreement with the pole and shape of the original shape. With simulated Gaia data, we are thus able to obtain overall shape of the object, but not the local shape details.

CONCLUSIONS

We have applied convex stochastic optimization and MCMC inversion methods to derive asteroid spins and shapes using simulated Gaia photometry. The original and inverted shapes are overall in good agreement. The local features are not reflected in the inverted shape. MCMC asteroid lightcurve inversion methods can potentially be applied to the forecoming asteroid photometric observations by the Gaia mission [6] or the lightcurves stored in Standard Asteroid Photometric Catalogue (http://asteroid.astro.helsinki.fi/).

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