A new three-parameter H,G_1,G_2 magnitude phase function for asteroids

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We have developed a new three-parameter H,G_1,G_2 magnitude phase function for asteroids. The phase function is aimed at replacing the currently adopted two-parameter H,G phase function. We show that H,G_1,G_2 produces better fits of available magnitude - phase curves of well-observed asteroids. We show also that the new system can be conveniently reduced to a two-parameter H,G_{12} magnitude phase function, which allows us to derive better estimates of the absolute magnitudes of asteroids for which poorly-sampled magnitude phase curves are available.

INTRODUCTION

Apparent position, motion, and brightness are the three basic pieces of information one can immediately derive for an asteroid based on a remote observation using an optical telescope. Measurements of position and motion are used to derive the orbits of the objects. From knowledge on the orbits, one can derive the corresponding distances of the object from the observer and from the Sun at any given epoch of observation. Based on this, it is possible to convert available measurements of apparent brightness into information on the intrinsic brightness of the object. On one hand, this operation consists of simply converting measurements of apparent brightness at a given epoch of observation into corresponding estimates of the brightness that would have been measured, if the object had been located at a fixed distance (usually assumed to be equal to 1 Astronomical Unit) from both the Sun and the observer. This can be easily done based on the trivial r^{-2} and Δ^{-2} dependence of the brightness, where r and Δ are the distances from the object to the Sun and to the observer, respectively. On the other hand, a more difficult problem is due to the fact that asteroids can be observed in a variety of illumination conditions, and the resulting brightness depends on these conditions. In particular, asteroids are brighter when seen close to the heliocentric opposition, that is when the so-called *phase angle*, namely the angle between the directions to the Sun and to the observer as seen from the asteroid, approaches zero. For increasing values of the phase angle, the objects tend to become increasingly fainter (using astronomical terminology, their magnitude increases). The change of magnitude as a function of phase angle is usually represented by the so-called phase curves. The increase of magnitude for

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increasing phase angle generally follows a nearly linear trend in a wide interval of phase angles, but close to opposition, at phase angles less than about $10°$, a non-linear brightness surge, usually named the *opposition effect*, takes place.

Based on the above considerations, the *absolute magnitude* of an asteroid is defined as the apparent brightness in standard V light that would be measured if the object was observed at a unit distance from both the Sun and the observer and at zero phase angle. Here, we do not deal with further complications due to fact that the objects are not spherical and then exhibit a periodic variation of brightness due to their rotation. The main point here is that the absolute magnitude is a very important parameter, since it is directly related to the size of an object and to its albedo (reflectance), which are fundamental physical properties.

Due to its importance from the point of view of physical studies for asteroids, the derivation of the absolute magnitude must be considered as a very important and delicate task. For this reason, the development of suitable magnitude phase functions is of outstanding importance for asteroid science. To address this issue, the International Astronomical Union (IAU) adopted in 1985 the so-called H,G system. The two parameters have the following meaning: H corresponds to the mean brightness, in Johnson V magnitude, at zero-degree phase angle, and corresponds thus to the absolute magnitude as defined above; G is the so-called slope parameter, which describes the general behavior of magnitude-phase curves. In practical terms, the slope parameter has been derived only for a tiny fraction of the known asteroids. This is due to the fact that, in general, only a handful of photometric observations, obtained at only a few phase angles, are available for any given asteroid, and this is insufficient to derive both H and G. In these cases, an assumed value of G, usually 0.15, is adopted [1]. Currently, all major catalogues of H values, with the only one notable exception of the AstDys database maintained at the University of Pisa, trace their values to the Minor Planet Center's database. Unfortunately, the accuracy of these values usually turns out to be poor, probably due to low-quality photometry used to obtain the H (and in a few cases, the G) values. As a consequence, asteroid V-band magnitudes predicted from the available H,G values are usually affected by significant errors. In particular, the objects tend to be very often significantly fainter than predicted [2, 3].

In this paper, we briefly summarize some recent results we have obtained concerning the development of a new three-parameter magnitude phase function that aims at replacing the two-parameter phase function that is currently adopted by IAU. An extensive explanation of our procedures and results is given in[4].

FROM H,G TO H,G_1,G_2

The H,G magnitude phase function was developed from efforts by several authors to model light-scattering phenomena in planetary regoliths. It has the following form for the reduced observed magnitude V:

$$V(\alpha) = H - 2.5 \log_{10}[(1 - G)\Phi_1(\alpha) + G\Phi_2(\alpha)], \tag{1}$$

where α is the phase angle, and $\Phi_1(\alpha)$ and $\Phi_2(\alpha)$ are two basis functions normalized at unity for $\alpha = 0°$. According to Eq. (1), the magnitude phase curve (hereinafter, simply "phase curve") of an object is described as the partitioning of the Φ_1 and Φ_2 functions in the ratio $(1 - G) : G$. In turn, the slope parameter G is scaled in such a way that it is close to 0 for steep phase curves, and close to 1 for shallow phase curves, but values outside this interval are not excluded *a priori*.

The explicit mathematical expressions for $\Phi_1(\alpha)$ and $\Phi_2(\alpha)$ can be found in [1, 4]. These two functions constituted the state of the art at the time of development of this photometric system. Their forms were suggested by theoretical models of light scattering that at that time did not include yet the phenomenon of coherent backscattering.

The H,G phase function still does a reasonably good job in fitting phase curves for many asteroids, especially in the region from $\sim 10°$ to $\sim 60°$. However, there are now some high-quality phase curves for which this is not really true, especially at phase angles in the region of the opposition brightness surge.

As a consequence of a thorough exercise to optimize $\Phi_1(\alpha)$ and $\Phi_2(\alpha)$ using stochastic optimization methods, we concluded that no "minor" revision of the H,G phase function can lead to a substantial improvement of the best fit to high-quality photometric phase data. This seems to be due to the intrinsic limits imposed by the choice of a linear two-parameter system using fixed basis functions. Therefore, we were obliged to conclude that a better fit of high-quality photometric data can only be obtained by adding an additional parameter to the photometric phase function. This is what we call our new H,G_1,G_2 magnitude phase function.

In the H,G_1,G_2 magnitude system for asteroids, the reduced observed magnitudes can be obtained from

$$\begin{aligned} 10^{-0.4V(\alpha)} &= a_1\Phi_1(\alpha) + a_2\Phi_2(\alpha) + a_3\Phi_3(\alpha) \\ &= 10^{-0.4H}[G_1\Phi_1(\alpha) + G_2\Phi_2(\alpha) + (1 - G_1 - G_2)\Phi_3(\alpha)], \end{aligned} \quad (2)$$

where the absolute magnitude H and the coefficients G_1 and G_2 are

$$H = -2.5\log_{10}(a_1 + a_2 + a_3), \quad G_1 = \frac{a_1}{a_1 + a_2 + a_3}, \quad G_2 = \frac{a_2}{a_1 + a_2 + a_3}. \quad (3)$$

The coefficients a_1, a_2, and a_3 are estimated from the observations by using the linear least-squares method. Thereafter, H,G_1, and G_2 follow from Eq. (3). As for the three basis functions, they must trivially satisfy the condition $\Phi_1(0) = \Phi_2(0) = \Phi_3(0) = 1$. Our idea was to construct a magnitude phase function consisting of an opposition-effect function Φ_3 and two linear basis functions Φ_1 and Φ_2. The derivation of their final explicit mathematical form by means of the stochastic optimization method is thoroughly explained in [4].

The results of this exercise were quite positive. In practically all analyzed cases of our sample, we obtained best-fit curves better, in terms of residuals, with respect to the corresponding H,G solutions. In several cases, the resulting best fits were significantly better. Again, all plots and Tables are given in [4]. The proposed three-parameter magnitude phase function is therefore much better in reproducing the behavior exhibited by well-observed, real objects. Such a conclusion might appear, however, as a speculative exercise in practice, if we think that for the vast majority of asteroids only a handful of (often poor) photometric data are available. In this situation, applying a three-parameter magnitude phase function would seem *a priori* a sterile exercise. Fortunately, the situation seems much different. The reason is that we found that, by considering the whole set of observed objects in our sample, a correlation exists between our derived G_1 and G_2 parameters. This fact can then be exploited, to express G_1 and G_2 as functions of a single G_{12} parameter, keeping the basis functions Φ_1, Φ_2, and Φ_3 fixed. It is then possible to fit the data by using a new two-parameter magnitude phase function that we call the H,G_{12} phase function. Once G_{12} is determined, the parameters of the three-parameter system H,G_1 and G_2 can be computed correspondingly.

In this way, we found a satisfactory fit of asteroid phase curves even when only a small number of observations are available (by artificially removing large numbers of observations from the phase curves in our sample). Some preliminary tests of the predictive power of the H,G_{12} approach are given in [4]. Figure 1 shows the nonlinear least-squares fit using the H,G_{12} phase function to the observations of asteroids (24) Themis and (44) Nysa. We obtain the following best-fit parameter values with 3-σ error estimates: for Themis, $H = 7.12 \pm 0.04$ mag, $G_{12} = 0.7 \pm 0.2$, $G_1 = 0.7 \pm 0.2$, and $G_2 = 0.1 \pm 0.1$; for Nysa, $H = 6.90 \pm 0.04$ mag, $G_{12} = 0.07 \pm 0.08$, $G_1 = 0.01 \pm 0.06$, and $G_2 = 0.69 \pm 0.07$.

The next step of our analysis will be precisely to investigate in more details the performances of this method in different situations that can be encountered in practice, including those that will correspond to the forthcoming Gaia and Pan-STARRS sky surveys. Taking into account the significant errors that affect the absolute magnitudes in current asteroid catalogs, we think that our proposed H,G_1,G_2 phase function can be an extremely useful tool to derive accurate absolute magnitudes from the huge amount of new data we can expect to come from the new-generation sky surveys from the ground and from space.

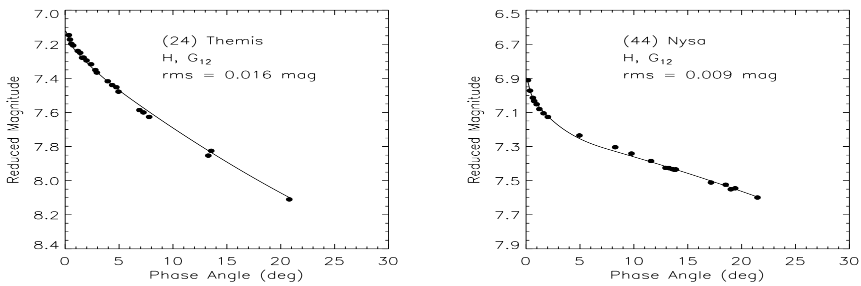

Figure 1. Nonlinear least-squares fits (solid lines) to the phase curves of the C-class asteroid (24) Themis (left; observations by Harris et al. [5]) and the E-class asteroid (44) Nysa (right; observations by Harris et al. [6]) using the two-parameter H, G_{12} magnitude phase function.

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