

# Polarizing efficiency of nonspherical scatterers of different structure

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By comparing porous multilayered spheroids and homogeneous ones with effective permittivity, we find that the ratio of the linear polarization degree of transmitted radiation to optical thickness of a medium containing partly aligned nonspherical inhomogeneous particles can strongly depend on their structure. We consider this effect in some detail and note that it occurs for any nonspherical scatterers with inclusions as well.

## INTRODUCTION

The optical properties of homogeneous nonspherical particles have been extensively modeled in the past 25 years (see, e.g., [1, 2]). In contrast, inhomogeneous nonspherical scatterers have been studied rather seldomly. Exceptions are fractal aggregates (see [3] and references therein) and particles with randomly distributed small inclusions for which effective medium approximations provide acceptable results (see, e.g., [4]).

In this paper we consider the linear polarization of radiation passing through an ensemble of partly aligned nonspherical inhomogeneous (porous) spheroids of different structure. Sect. 2 describes our models of inhomogeneous scatterers and computational methods used. Sect. 3 presents results of calculations and their discussion.

## MODELS AND METHODS

We compare the optical properties of porous spheroids of three kinds (see Fig. 1):

**Model I** — Scatterers with randomly distributed tiny inclusions approximated by homogeneous particles with the refractive index given by an effective medium theory (EMT).

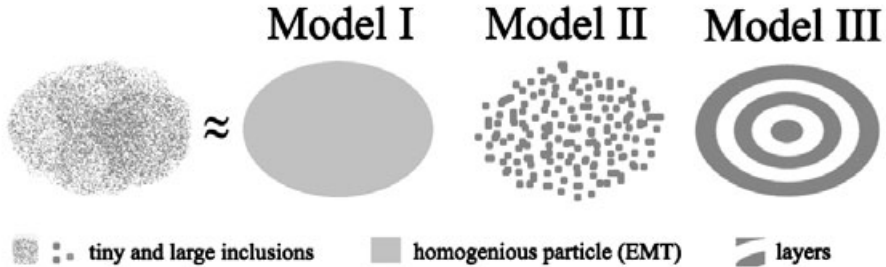
**Model II** — Scatterers with randomly distributed large inclusions modeled by applying the standard discrete-dipole approximation (DDA) approach.

**Model III** — Scatterers with many cyclically repeating layers considered according to [5]. Note that when the number of layers becomes large, the scattering characteristics tends to depend only on the material volume fractions and tends to be independent of the order and number of layers. The optical properties of multilayered particles were obtained by the generalized separation of variables method (SVM) with a spherical basis [6].

Thus, the *general parameters* are the spheroid *aspect ratio*  $a/b$  (and the type: prolate/oblate), *size parameter*  $x_V = 2\pi r_V/\lambda$ , where  $r_V$  is the radius of a sphere whose volume is equal to that of the spheroid and  $\lambda$  the wavelength, *orientation angle*  $\alpha$  between the particle symmetry axis and the incident radiation wavevector, *porosity*  $\mathcal{P}$  being a fraction of the particle volume filled by material with a *refractive index*  $m$ .

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*Additional parameters* are, for the model II, the *halfwidth of cubic inclusions*  $r$  (in units of the interdipole distance) and, for the model III, the *number of layers*  $L$ , *aspect ratios* of the external boundaries  $(a/b)_i$ , *volume fractions*  $V_i/V_{\text{tot}}$  ( $V_{\text{tot}}$  is the particle volume) and *refractive indices*  $m_i$  of the layers ( $i = 1, 2, \dots, L$ ).



**Figure 1.** Cross-sections of three model spheroids considered.

## NUMERICAL RESULTS AND DISCUSSION

We consider extinction and linear polarization of radiation passing through a medium populated by single size porous spheroids in a picket-fence orientation.

Dimensionless extinction cross-sections of spheroids are calculated for two kinds of incident plane wave polarizations:  $Q_{\text{ext}}^{\text{TE}}$ ,  $Q_{\text{ext}}^{\text{TM}}$ . For an unpolarized radiation, the *extinction and polarization cross-sections* of the particles are  $Q_{\text{ext}} = (Q_{\text{ext}}^{\text{TM}} + Q_{\text{ext}}^{\text{TE}})/2$  and  $Q_{\text{pol}} = (Q_{\text{ext}}^{\text{TM}} - Q_{\text{ext}}^{\text{TE}})/2$ , respectively.

We compare the *polarizing efficiency*  $P = |Q_{\text{pol}}|/Q_{\text{ext}}$  (equals to the ratio of the linear polarization degree of transmitted radiation to optical thickness  $p/\tau$ ) calculated for spheroids of the same parameters  $a/b$ ,  $x_V$ ,  $\alpha$ ,  $\mathcal{P}$  and  $m$ , but of different structure (the models I--III).

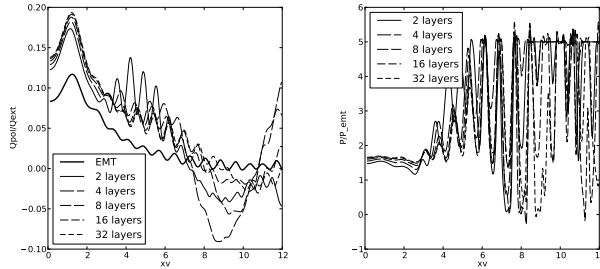
We find that the ratio  $Q_{\text{pol}}/Q_{\text{ext}}$  for porous layered spheroids (model III) systematically differs from that for the corresponding spheroids with an effective permittivity (model I) (see the left panel of Fig. 2), i.e., the *polarizing efficiency of inhomogeneous spheroids may essentially depend on their structure*.

The strength of this effect is more clearly illustrated in the right panel of Fig. 2, where we plot the ratio of polarizing efficiencies for the models III and I as  $P/P_{\text{emt}}$ . Note that this ratio exceeds 1.5 for *all particle sizes* and can be as large as 2--5.

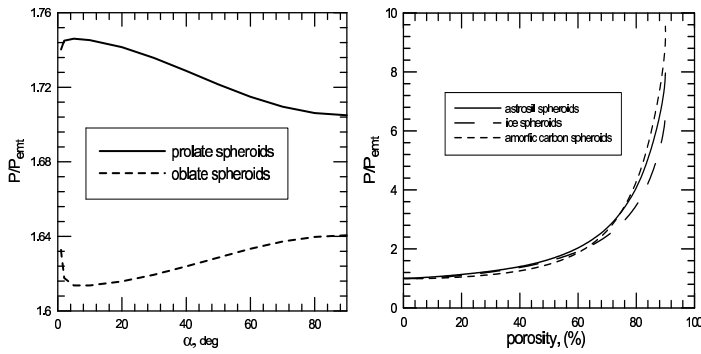
The effect depends weakly on the orientation of both the prolate and oblate spheroids but is strongly affected by the particle porosity  $\mathcal{P}$  and composition for  $\mathcal{P} > 70\%$  (see Fig. 3).

It should be especially important for interpretation of the interstellar extinction and polarization phenomena within the existing porous models of cosmic dust grains. That is why we use refractive index values typical of astronomical silicate, ice, and amorphous carbon in the visual region. The left panel of Fig. 4 gives the ratio of the polarization degree of transmitted radiation to optical thickness  $p/\tau$  for spheroids of different aspect ratio  $a/b$  and porosity  $\mathcal{P}$ . Note that if an observed value of  $p/\tau$  was earlier fitted by using homogeneous (model I) spheroids of a ratio  $a/b$ , now this value can be obtained for porous particles of essentially lower  $a/b$ .

According to [5], the mathematical model of multilayered particles can be useful in prac-



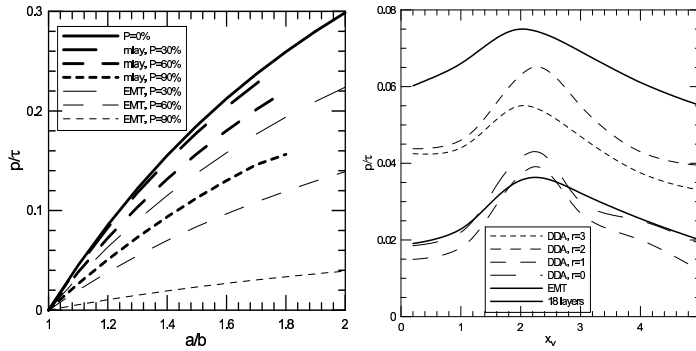
**Figure 2.** *Left panel:* Extinction cross-section ratio  $Q_{\text{pol}}/Q_{\text{ext}}$  as a function of the size parameter  $x_V$  for the model I (homogeneous) and different types of model III (layered with  $L = 2, 4, 8, 16, 32$ ) porous icy prolate spheroids. *Right panel:* Polarizing efficiency for the porous layered (model III) spheroids  $P$  normalized to that for the corresponding homogeneous (model I) particles  $P_{\text{emt}}$ . Other parameters are  $m = 1.3$ ,  $\alpha = 45^\circ$ ,  $\mathcal{P}=50\%$ ,  $(a/b)_i = 1.4$  and  $V_i/V_{\text{tot}} = L^{-1}$  for  $i = 1, 2, \dots, L$ .



**Figure 3.** Ratio of the polarizing efficiencies for layered (model III) and homogeneous (model I) spheroids  $P/P_{\text{emt}}$  as a function of particle parameters. *Left panel:* Different orientation of porous prolate and oblate spheroids ( $m = 1.7 - 0.03i$ ,  $x_V = 1$ ,  $\mathcal{P}=50\%$ ). *Right panel:* Prolate spheroids of different porosity and composition ( $m = 1.3, 1.7 - 0.03i$  and  $1.98 - 0.23i$  for ice, astrotil, and amorphous carbon, respectively,  $x_V = 3$ ,  $\alpha = 90^\circ$ ). Other parameters are  $L = 20$ ,  $(a/b)_i = 1.5$  and  $V_i/V_{\text{tot}} = L^{-1}$  for  $i = 1, 2, \dots, L$ .

tice. We confirm this statement by comparing the polarizing efficiency of porous multilayered (model III) and homogeneous (model I) spheroids and (quasi)spheroids with inclusions of different size (model II --- see the right panel of Fig. 4). One can see that the ratio  $p/\tau$  for scatterers with small inclusions ( $r = 0$  and  $1$ ) is close to the EMT ratio, while the values of  $p/\tau$  obtained for the case of larger inclusions ( $r = 2$  and  $3$ ) tend to those given by the multilayered model; i.e., by comparing two simple computational models III and I, one can reveal possible particle structure effects for scatterers with inclusions.

To conclude, we have found that the ratio of the linear polarization degree of transmitted radiation to optical thickness of a medium containing partly aligned nonspherical porous spheroids can strongly depend on their structure, i.e., distribution of materials inside the scatterers. The effect is especially evident for highly porous particles, but should occur for any nonspherical scatterers with large inclusions when refractive indices of the matrix and



**Figure 4.** Polarizing efficiency  $p/\tau$  for structurally different porous oblate spheroids. *Left panel:* Comparison of the models III and I for different aspect ratios  $a/b$  and porosity  $\mathcal{P}$  ( $m = 1.7 - 0.03i$ ,  $x_V = 1$ ). *Right panel:* Comparison of the models III, I, and different models II for size parameters  $x_V < 5$  ( $a/b = 1.4$ ,  $\mathcal{P} = 90\%$ , layers and inclusions of two materials with  $m_1 = 1.7 - 0.03i$  and  $m_2 = 1.98 - 0.23i$ ). Other parameters are  $\alpha = 90^\circ$ ,  $L = 18$ ,  $(a/b)_i = a/b$  and  $V_i/V_{\text{tot}} = L^{-1}$  for  $i = 1, 2, \dots, L$ .

inclusion materials essentially differ. So, the results are general and should have different applications besides interstellar extinction and polarization interpretation mentioned.

**Acknowledgements:** The work was supported by the grants RFFI 10-02-00593, NTP 2.1.1/665 and NSh 1318.2008.2.

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