Measurement of block-diagonal scattering matrix

S. Savenkov*[,1], R. Muttiah[2] , E. Oberemok[1], and A. Klimov[1]

[1] *Radiophysics Department, Kiev Taras Shevchenko University, Kiev, 01 033 Ukraine.*
[2]*Alan Plummer Associates, Inc., 1320 S. University Dr. #300, Fort Worth, Texas 76107, USA.*

The problem of measurement of the block-diagonal scattering matrix is addressed. It has been shown that non-zero elements of the scattering matrix with block-diagonal structure can be measured using only two polarizations of input radiation. Optimal pairs of these polarizations are derived. Utilization of the optimal input polarization results in half the measurement time and a decrease in measurement errors of approximately 30%.

INTRODUCTION

The block-diagonal scattering matrix of the form

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{pmatrix}, \tag{1}$$

plays a key role in many light-scattering problems. The structure of **M** in Eq. (1) can be the result of symmetry of a single particle or a collection of particles in multiple scattering [1,2] and by the illumination-observation geometry for backward [3] and forward [4,5] scattering.

Scattering matrix **M** was used in the study of the optical characteristics of oceanic water [6,7]; of an ensemble of identical, but randomly oriented fractal particles [8]; of dense spherical particle suspensions in multiple-scattering case [9]; of the multiple scattering of light by an ice cloud consisting of nonspherical ice crystals [10]; of polydisperse, randomly oriented ice crystals modeled by finite circular cylinders with different size distributions [11]; for characterizing cylindrically shaped radially inhomogeneous particles [12]; small spherical particles (diameters range from 0.2 to 1.5 μm) sparsely seeded on a surface of crystalline silicon *c*-Si wafer [13]; for measurements of the complex refractive index of isotropic materials as matrices of isotropic and ideal metal mirror reflections [14]; in developing a symmetric three-term product decomposition of a Mueller-Jones matrix [15]; in the very general and important cases of (i) randomly oriented particles with a plane of symmetry [16] and (ii) with equal numbers of particles and their mirror particles [17].

Non-zero elements of the matrix **M** can be considered as the corresponding incomplete scattering matrices. Our main concern here is the utilization of this fact in the experimental

* Corresponding author: Sergey N. Savenkov (sns@univ.kiev.ua)

determination of non-zero elements of matrix **M** to decrease the error and time of measurements.

MEASUREMENT OF BLOCK-DIAGONAL SCATTERING MATRIX

The most appropriate measurement strategy of the block-diagonal scattering matrix **M** in Eq. (1) is the so-called time-sequential measurement strategy [18]. The measurement equation [18] of the time-sequential strategy takes the form

$$\begin{aligned}&\left(m_{11}r_1^1 + m_{12}r_2^1 \quad m_{11}r_1^2 + m_{12}r_2^2 \quad m_{11}r_1^3 + m_{12}r_2^3 \quad \cdot \quad \cdot \quad \cdot \quad m_{43}r_3^4 + m_{44}r_4^4\right)^T = \\ &\left(s_1^1 \quad s_1^2 \quad s_1^3 \quad s_1^4 \quad \cdot \quad \cdot \quad s_4^4\right)^T.\end{aligned} \tag{2}$$

Here r_i^k, s_i^k are the i-th parameters of the k-th Stokes vectors of input and output radiation. From Eq. (2) it can be deduced that the non-zero matrix elements can be measured by using only two input polarizations. In other words, Eq. (2) can be reduced to two independent subsystems of equations relative to the non-zero elements of **M** with the following characteristics matrices

$$\mathbf{V}_1 = \begin{pmatrix} r_1^1 & r_2^1 \\ r_1^2 & r_2^2 \end{pmatrix}, \quad \text{and} \quad \mathbf{V}_2 = \begin{pmatrix} r_3^1 & r_4^1 \\ r_3^2 & r_4^2 \end{pmatrix}. \tag{3}$$

It can be seen that the rows of the characteristics matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ are formed by Stokes parameters of two polarizations of input radiation. The most important question resulting from Eq. (3) is what two polarizations should one use to measure the non-zero elements of the matrix **M**?

CHOICE OF INPUT POLARIZATIONS

If the values of parameters r_i^k, s_i^k in Eq. (2) are known accurately, i.e. without measurement errors, then the answer to this question is as follows: any two polarizations giving $\det(\mathbf{V}_1) \neq 0$ and $\det(\mathbf{V}_2) \neq 0$. However, this is not the case in the presence of measurement errors. In this case we use the condition number method [19]. Evidently, the connection between the matrices in Eq. (3) dictates that the values of Stokes parameters of input polarizations r_i^k should minimize the condition numbers of both characteristics matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ in Eq.(3) simultaneously. This can be realized by minimization of the condition number of the product of the matrices $\mathbf{V}_1\mathbf{V}_2=\mathbf{V}$. In this case the condition number is a function of four variables: two azimuths and two ellipticities of two input polarizations.

Analysis shows that the minimal value of condition number is $cond(\mathbf{V}) = 2$, which corresponds to $cond(\mathbf{V}_1)\big|_{\min} = cond(\mathbf{V}_2)\big|_{\min} = 3.18$, and this solution is not unique. Graphically the dependences of the condition number $cond(\mathbf{V})$ on values of ellipticities $\varepsilon_{1,2}$ and azimuths $\theta_{1,2}$ of input polarizations are presented in Fig.1.

Dependences presented in Fig.1 correspond to the following pair of optimal input polarizations $\varepsilon_1 = 14.7°$, $\varepsilon_2 = 37.6°$, $\theta_1 = 79.8°$, $\theta_2 = -42.5°$. The explicit form of the characteristics matrices Eq.(3) in this case are

$$\mathbf{V}_1 = \begin{pmatrix} 1 & -0.8164 \\ 1 & 0.0218 \end{pmatrix}; \quad \mathbf{V}_2 = \begin{pmatrix} 0.3043 & 0.4907 \\ -0.2539 & 0.9670 \end{pmatrix}. \qquad (4)$$

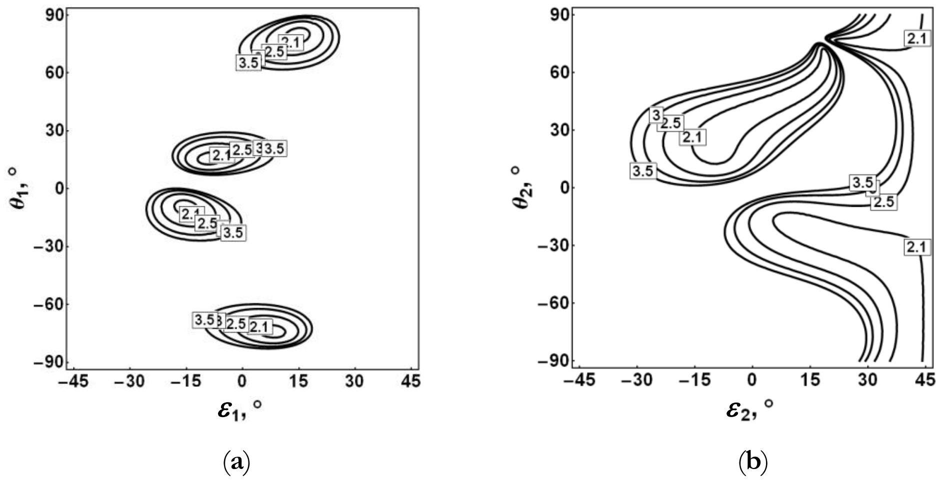

Fig.1. Dependences of condition number $cond(\mathbf{V})$ on values of ellipticities $\varepsilon_{1,2}$ and azimuths $\theta_{1,2}$ for first (**a**) and second (**b**) polarization of input radiation (other polarizations in both cases is fixed and optimal).

CONCLUSIONS

We have demonstrated that non-zero elements of the block-diagonal scattering matrix **M** can be measured using only two polarizations of input radiation (see Fig. 1) and derived optimal pairs of these polarizations. Note that in practice the measurements with two input polarizations should be foreshadowed by calibration measurements with four input polarizations, which determine whether the scattering matrix has exactly block-diagonal form. If it is the case, then the utilization of derived optimal input polarizations in Fig. 1 results in half the measurement time and approximately a 30 % decrease of measurement errors, with all else being equal.

REFERENCES

[1] H.C . van de Hulst. *Light Scattering by Small Particles*. Wiley, New York (1957).

[2] M.I. Mishchenko and L.D. Travis. Polarization and depolarization of light. In: *Light Scattering from Microstructures*. F. Moreno and F. González (eds.). Springer-Verlag (2000).

[3] E. Zubko, Y.G. Shkuratov, and G. Videen. Coherent backscattering effect for non-zero elements of Mueller matrix of discrete media at different illumination-observation geometries. J. Quant. Spectrosc. Radiat. Transfer **89** (2004).

[4] S.N. Savenkov, R.S. Muttiah, K.E. Yushtin, and S.A. Volchkov. Mueller-matrix model of an inhomogeneous, linear, birefringent medium: single scattering case. J. Quant. Spectrosc. Radiat. Transfer **106** (2007).

[5] S.N. Savenkov, V.I. Grygoruk, S.M. Ranjan, Y.A. Oberemok, K.E. Yushtin, and V.V. Yakubchak. Effective dichroism in forward scattering by inhomogeneous birefringent medium. J. Quant. Spectrosc. Radiat. Transfer **110** (2008).

[6] K. J. Voss and E. S. Fry. Measurement of the Mueller matrix for ocean water. Appl. Optics **23** (1984).

[7] A.A. Kokhanovsky. Parameterization of the Mueller matrix of oceanic waters. J. Geophys. Res. **108** (2003).

[8] A.A. Kokhanovsky. Optical properties of irregularly shaped particles. J. Phys. D: Appl. Phys. **36** (2003).

[9] B. Kaplan, G. Ledanois, and B. Drevillon. Mueller matrix of dense polystyrene latex sphere suspensions: measurements and Monte Carlo simulation. Appl. Opt. **40** (2001).

[10] R. Lawless, Y. Xie, P. Yang, G.W. Kattawar, and I. Laszlo. Polarization and effective Mueller matrix for multiple scattering of light by nonspherical ice crystals. Opt. Express **14** (2006).

[11] L. Xu, J. Ding, and A. Cheng. Scattering matrix of infrared radiation by ice finite circular cylinders. Appl. Optics **41** (2002).

[12] S. Manickavasagam and P. Menguc. Scattering-matrix elements of coated infinite-length cylinders. Appl. Optics **37** (1998).

[13] B. Kaplan and B. Drevillon. Mueller matrix measurements of small spherical particles deposited on a *c*-Si wafer. Appl. Optics **41** (2002).

[14] L. Deibler and M. Smith. Measurement of the complex refractive index of isotropic materials with Mueller matrix polarimetry. Appl. Optics **40** (2001).

[15] R. Ossikovski. Interpretation of nondepolarizing Mueller matrices based on singular-value decomposition. J. Opt. Soc. Am. A **25** (2008).

[16] J. Hovenier and C. van der Mee. Basic relationships for matrices describing scattering by small particles. In: *Light Scattering by Nonspherical Particles*. M. Mishchenko, J. Hovenier, and L. Travis (eds.). Academic Press, NY (2000).

[17] M.I. Mishchenko, L.D. Travis, and A.A Lacis. *Multiple scattering of light by particles*. Cambridge Univ. Press, Cambridge (2006).

[18] R.A. Chipman. Polarimetry. In: *Handbook of Optics*. Vol. II, McGraw Hill (1995).

[19] R. Horn and Ch. Johnson. *Matrix Analysis*. Cambridge Univ. Press, London (1986).