# Modeling radar backscattering from melting snowflakes at C-band using DDA and TMM

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We model radar backscattering in the C-band from fluffy snowflakes at early stages of melting using both the discrete-dipole approximation (DDA) and the *T*-matrix method (TMM). DDA approximates the particle as a cubic lattice of dipoles, while TMM is an exact method for nonspherical particles. To simulate falling snowflakes at early melting, the particles are modeled as oriented oblate spheroids and have melted only at the bottom part. We use two different dipole distributions for DDA; random single dipoles and dipole groups consisting of  $4\times 4\times 4$  dipoles. We find that even a very fluffy particle shows resonance features for most radar parameters. Using the Maxwell-Garnett effective medium approximation in TMM computations seems to match DDA well for all sizes studied. Small amount of inhomogeneously distributed water has a negligible effect on most radar parameters for both DDA and TMM. Only the specific differential phase  $K_{DP}$  shows clear deviation between DDA and TMM. Using dipole groups instead of single dipoles to approximate a fluffy snowflake seems to effect  $K_{DP}$  the most.

### INTRODUCTION

Uncertainty in modeling of scattering properties of wet snow particles is one of the major error sources in melting layer modeling [1]. Typically, wet snow particles are modeled using spherical or spheroidal shapes with a dielectric constant defined by an effectivemedium approximation [2]. It is, however, known that melting hydrometeors represent non-homogeneous scatterers. Mitra et al. [3] have presented results of wind-tunnel observations of snow-melting behavior. Based on those observations, several melting stages were identified. In this study, we analyze scattering properties of the melting snow particles at the initial stages of melting. At this stage, small water droplets, with diameters of tens of microns, are formed at the periphery of a snow flake. These droplets are mainly concentrated at the lower part of the particle. This results in an inhomogeneous distribution of water in the particle. Given the sensitivity of dual-polarization radar observations on the shape and dielectric properties of hydrometeors, it is important to analyze whether such particle inhomogeneities cause observable radar signatures.

In our previous study [4], we modeled radar backscattering from simple shapes, like spheres, spheroids, and clusters of spheres. We used four different water contents for the coated spheres and the clusters. We used both DDA and TMM for modeling. The results showed that DDA agreed well with the exact solutions for homogeneous particles but, for water-coated particles, the grid should be large enough in order to preserve skin depth. Using effective-medium approximations in the Rayleigh approximation produced significant errors when compared to the results for the clusters of spheres. It was also noted that using the

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filtered coupled dipole (FCD) as polarizability in the DDA modeling effectively doubled the accuracy for water particles. We have adopted FCD in our DDA computations.

## MODELING

For the modeling, we use an oblate, spheroidal shape to represent falling snowflakes. The outer shape is used as a spheroidal surface envelope for the fluffy interior, which is composed of an array of discrete ice dipoles in uniform random positions. We use two different dipole distributions; random single dipoles (case 1) and dipoles that are clumped into  $4 \times 4 \times 4$  groups (case 2) in order to check how the skin depth is affected. The total number of dipoles is practically the same for these two cases. To simulate the early melting process, the ice dipoles near the bottom surface of the particle are changed to water randomly using Gaussian statistics. We have prohibited any melting above the equator of the particle. In Fig. 1, we show example particles for case 1 (left panel) and for case 2 (right panel).

Scattering computations are conducted using the DDA code by [5] and the TMM code by [6]. For the latter, the Maxwell-Garnett effective-medium approximation is used to obtain the corresponding refractive indices. In the computations, we use diameters D = 1.0-50.0mm and two different water contents (relative to mass): 0 %, and 1 %. The aspect ratio is 0.6, which is a typical value observed for snowflakes [7]. We model in the C-band (5.6 GHz), so the corresponding equal-volume-sphere size parameters vary between 0.05 and 2.5. We used size-dependent density for the snowflakes  $\rho = 0.15D^{-1}$  (D in mm) according to [1]. Particles are horizontally oriented, so the incident direction is normal to the rotational axis. The scattering matrices are averaged over the horizontal orientation, around the symmetry axis. In addition, we average over two incident polarizations, parallel and perpendicular to the symmetry axis.

We plot the horizontal reflectivity  $Z_h = C(k,m)n_0|S_{11}|^2$ , the differential reflectivity  $Z_{DR} = |S_{22}|^2/|S_{11}|^2$ , the argument of the copolarized correlation coefficient  $\delta_{hv} = arg(S_{22}S_{11}^*)$ , and the specific differential phase  $K_{DP} = -\frac{2\pi}{k}n_0Re[S_{22}(\mathbf{n}, \mathbf{n})+S_{11}(\mathbf{n}, \mathbf{n})]$  as a function of size (k is the wavenumber,  $n_0$  the number density, C(k, m) a constant factor, and **S** the amplitude scattering matrix in the backscattering-alignment convention). See, e.g., [8] for more detailed definitions.

#### RESULTS

Figs. 2–3 show  $Z_h$  (top left panel),  $Z_{DR}$  (top right panel),  $\delta_{hv}$  (bottom left panel), and  $K_{DP}$  (bottom right panel) for both TMM and the DDA cases.

Fig. 2 shows the radar parameters for the 0 % water content case. For  $Z_h$ , there seems to be a 1–2 dB systematic difference between DDA and TMM. It is not yet clear, where the difference originates from. There is a strong resonance peak at D = 38.0 mm, which can also be seen for both DDA cases. This indicates that even a very fluffy particle preserves some resonance features typical for symmetric solid particles.  $Z_{DR}$  and  $\delta_{hv}$  show some variation between the DDA cases, but these are negligible. Both parameters show signs of the resonance peak. For  $K_{DP}$ , both DDA cases differ from TMM, with case 2 showing larger, and case 1 smaller, values than TMM.

Fig. 3 shows the radar parameters for the 1 % water content case. Relatively small

amount of water has a negligible effect, when compared to the dry case. Overall, TMM computations together with the Maxwell-Garnett effective medium approximation matches DDA well for all sizes studied. Small amount of inhomogeneously distributed water has a negligible effect on most radar parameters for both DDA and TMM. Only  $K_{DP}$  shows clear deviation between DDA and TMM. Using dipole groups instead of single dipoles to approximate a fluffy snowflake seems to effect  $K_{DP}$  the most.

In the future, we will continue this study using larger water contents, and also study the effect of canting, i.e., deviations from the horizontal orientation due to air drag, on radar parameters.



**Figure 1**. Sample shapes for the modeled particles with the density  $\rho = 0.04$  and aspect ratio 0.6. The single dipole case is on the left and the dipole group case on the right.

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**Figure 2**.  $Z_h$ ,  $Z_{DR}$ ,  $\delta_{hv}$ , and  $K_{DP}$  as a function of particle diameter in millimeters for the 0 % water content case. TMM computations are in thin solid lines, the DDA case for single dipoles is in thick dashed line, and the DDA case for the dipole groups is in thick solid line.



Figure 3. Same as in Fig. 2, but for the 1 % water content case.