Numerically-exact computer modeling of light scattering by random absorbing media

J. M. Dlugach*,[1], M. I. Mishchenko[2], and L. Liu[2]

[1]*Main Astronomical Observatory of the National Academy of Sciences of Ukraine, 03680, Kyiv, Ukraine.*
[2]*NASA Goddard Institute for Space Studies, NY 10025, New York, U.S.A.*

We employ the superposition *T*-matrix method to perform computations of electromagnetic scattering by a volume of discrete random medium densely filled with spherical particles with the real part of the refractive index m_R = 1.31, 1.5 and the imaginary part m_I = 0, 0.01, 0.1, 0.3. Our computations show that increasing the value of m_I can both decrease and increase manifestations of the coherent backscattering (CB) effect, dependent on the values of the particle packing density and the real part of the refractive index.

INTRODUCTION

In our previous publications [1–3], by using the numerically-exact solution of the Maxwell equations we have studied electromagnetic scattering by a volume of statistically homogeneous discrete random medium of various sizes, consisting of different numbers of non-absorbing particles. In particular, we have shown that all backscattering effects predicted by the low-density theory of CB (see, e.g., [4] and references therein) also take place in the case of a densely packed medium. In this paper, we extend our numerically-exact computer modeling to the case of a medium composed of absorbing particles and analyze the effect of absorption on scattering characteristics of a medium composed of different numbers of particles with different refractive indices. Note that earlier in [5], on the basis of computations performed for m_R = 1.32 and particle packing density $\tilde{\rho}$ = 22%, it was concluded that increasing absorption diminishes such optical effects as depolarization and CB.

METHODOLOGY

Our model of particulate random medium is a spherical volume of radius R filled with N identical non-overlapping spherical particles of radius r. The size parameter of the volume is kR, and the particle size parameter is kr, where k is the wave number. To model the complete randomness of particle positions within the spherical volume, we follow the approach adopted in [1–3]. This approach yields an infinite continuous set of random realizations of the scattering volume and allows us to use the highly efficient orientation-averaging technique afforded by the superposition *T*-matrix method [6]. The latter

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[*] Corresponding author: Janna Dlugach (dl@mao.kiev.ua)

represents a direct, numerically-exact computer solver of the macroscopic Maxwell equations for a multi-sphere group (see, e.g., [7]).

We assume that the random particulate volume is illuminated by a quasi-monochromatic beam of light and is observed from a distant point. Using the scattering plane for reference allows us to define the relation between the Stokes parameters of the incident ("inc") and scattered ("sca") light in terms of the normalized scattering matrix of the entire volume [4, 8]:

$$\begin{bmatrix} I^{sca} \\ Q^{sca} \\ U^{sca} \\ V^{sca} \end{bmatrix} \propto \begin{bmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{bmatrix} \begin{bmatrix} I^{inc} \\ Q^{inc} \\ U^{inc} \\ V^{inc} \end{bmatrix}, \tag{1}$$

where θ is the scattering angle. The zeros denote scattering matrix elements negligibly small (in the absolute sense) relative to the other elements at the same scattering angle.

Elements of the scattering matrix define specific properties of the scattered light corresponding to different states of polarization of the incident light. If the incident light is unpolarized, then the element $a_1(\theta)$, called the phase function, describes the angular distribution of the scattered intensity, while the ratio $-b_1(\theta)/a_1(\theta)$ gives the corresponding degree of linear polarization. If the incident light is polarized linearly in the scattering plane, then the linear polarization ratio $\mu_L=(a_1(\theta) - a_2(\theta))/(a_1(\theta) + 2b_1(\theta) + a_2(\theta))$. If the incident light is polarized circularly in the counterclockwise direction when looking in the direction of propagation, then the circular polarization ratio $\mu_C = (a_1(\theta) + a_4(\theta))/(a_1(\theta) - a_4(\theta))$.

NUMERICAL RESULTS

Some of the results of our extensive computations are presented in Figs. 1 and 2. Here, we limit ourselves to analyzing only the data obtained in the range of large scattering angles $150° \leq \theta \leq 180°$. The computations have been performed for a scattering volume of the size parameter kR =30 filled with spherical particles with a complex refractive index $m = m_R + \mathrm{i}m_I$, where $m_R = 1.31$ (Fig. 1), 1.5 (Fig. 2), and $m_I = 0, 0.01, 0.1, 0.3$. The values of the particle size parameter were adopted to be $kr = 2$ ($m_R = 1.31$) and $kr = 1.76$ ($m_R = 1.5$). The point is that in [2, 3] we have obtained that just for these values of kr the single scattering polarization has a wide horizontal "shelf" of near-zero values extending from $\theta = 150°$ up to 180°. But with growing N, the polarization $-b_1(\theta)/a_1(\theta)$ develops a pronounced minimum caused by the increasing contribution of multiple scattering. Such behavior of polarization is caused by the effect of CB, and the question is how the absorption can influence the behavior of the negative branch of polarization. In the case of $m_R = 1.31$, the number of constituent particles N was varied from 100 up to 400 what corresponds the variation of the packing density $\tilde{\rho}$ from 3.6% up to 14.6%, and for $m_R = 1.5$, N varies from 100 ($\tilde{\rho}$ = 2.4%) up to 600 ($\tilde{\rho}$ = 14.5%).

From Fig. 1 we see that in the range $\theta > 170°$ the backscattering peak in the normalized scattered intensity $a_1(\theta)/a_1(180°)$, caused by the effect of CB, decreases with increasing the absorption, and the angular widths of these peaks increase with increasing absorption. A somewhat different situation is seen in Fig. 2. Specifically, in the case of N = 100 ($\tilde{\rho}$ = 2.4%), increasing m_I from 0 up to 0.1 results in increasing intensity backscattering peak and decreasing its angular width. Then, when m_I = 0.3, the backscattering peak decreases and practically does not differ from that in the case of m_I = 0, 0.01. In the case of N=600 ($\tilde{\rho}$ = 14.5%), the intensity backscattering peak decreases with increasing absorption monotonously. Besides, note that when m_I = 0.3 the backscattering enhancement is still sufficiently strong for both values of N, which is indicative of a significant contribution of multiple scattering for m_R = 1.5 even in the case of such strong absorption.

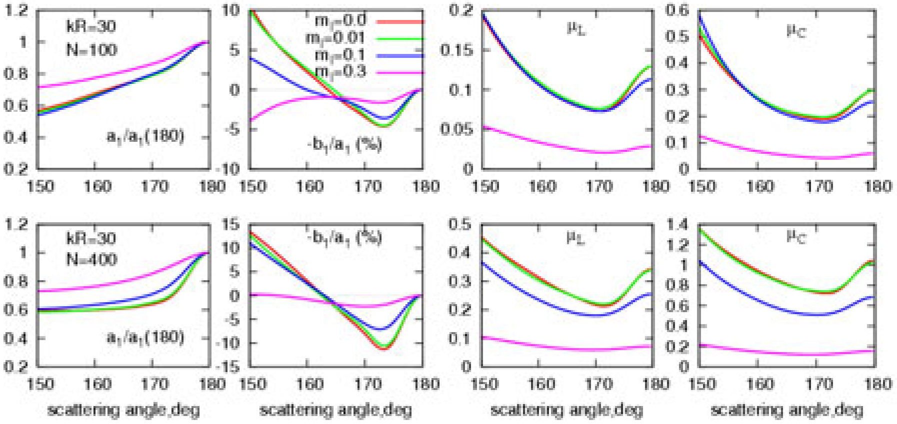

Figure 1. Scattering characteristics of a kR = 30 scattering volume randomly filled with N identical spherical particles of kr = 2.0, m_R = 1.31, and $0 \le m_I \le 0.3$. The upper row: N=100 (the packing density $\tilde{\rho}$ = 3.6%), the bottom row: N=400 ($\tilde{\rho}$ = 14.6%).

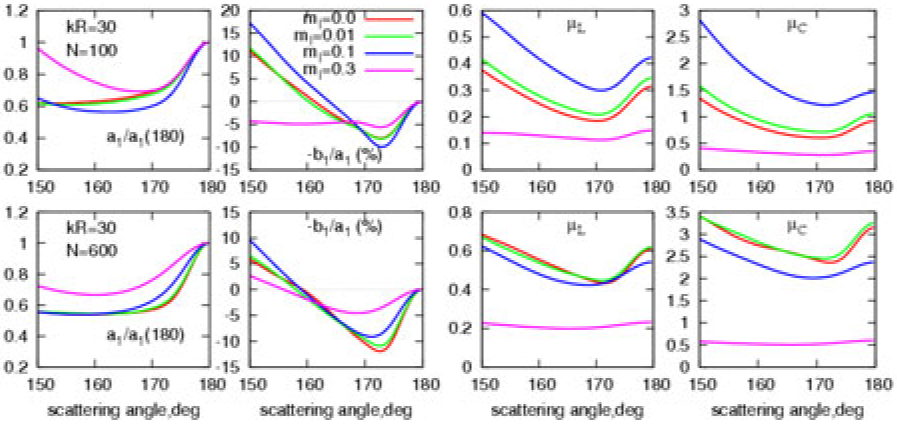

Figure 2. Scattering characteristics of a kR = 30 scattering volume randomly filled with N identical spherical particles of kr = 1.76, m_R = 1.5, and $0 \le m_I \le 0.3$. In the upper row, N = 100 (the packing density $\tilde{\rho}$ = 2.4%). In the bottom row, N=600 ($\tilde{\rho}$ = 14.5%).

In the case of $m_R = 1.31$ (Fig. 1), the minimum of polarization $-b_1(\theta)/a_1(\theta)$ monotonously decreases with increasing m_I, and when $m_I = 0.3$, the curve of polarization changes its shape. Note that for $N = 100$, the asymmetry of the polarization curve increases with increasing m_I up to 0.1, but if $N = 400$ then the position of the inversion point does not change. Fig. 2 demonstrates that in the case of $m_R = 1.5$ and $N = 100$, the behavior of the value of minimum polarization (similar for the behavior of the intensity) is not monotonous with increasing absorption. Firstly, increasing m_I from 0 up to 0.1 results in increasing the maximum absolute value of negative polarization, and, when $m_I = 0.3$, the absolute value of negative polarization decreases. For the case of $N = 600$, the depth of the polarization minimum decreases monotonously with increasing absorption.

Finally, from Fig. 1 we see that increasing absorption monotonously diminishes the values of the linear μ_L and the circular μ_C polarization ratios, and when $m_I = 0.3$, depolarization of the medium approaches zero. Again, the results of computations of μ_L and μ_C performed for $m_R = 1.5$ and $N = 100$ and shown in Fig. 2, demonstrate a significant increase of these values with increasing m_I from 0 up to 0.1.

Thus, we can conclude that increasing absorption can both enhance and suppress manifestations of the CB effect depending on the particle packing density and the real part of the refractive index. It is necessary to note that in [9] it has been shown that absorbing Rayleigh scatterers can yield more pronounced enhancement factors and polarization surges than nonabsorbing Rayleigh scatterers.

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